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MATHEMATICAL MODELING OF TRANSMISSION START WITH AN ASYNCHRONOUS ELECTRIC MOTOR

Abstract. *The most difficult moment in the work with an asynchronous motor is the launch. And the more powerful drive is the more difficult launch. This is due to certain features of the asynchronous motors: a limited starting torque and starting throws of the current of the stator motor chain.*
The mathematical modeling of oscillating process of actuation of the actuator with an asynchronous motor, which includes an elastic coupling with nonlinear mechanical feedback, is carried out. The influence of the type of elastic characteristics of the coupling on the magnitude of the amplitude and frequency of the oscillation process and its time was studied. A single-mass rotational system model was used for the studies. According to the Runge-Kutta method, the oscillation processes of starting the transmission of a machine unit with an induction motor were investigated. To determine the coefficient of vibration isolation, a system with an elastic coupling having a linear elastic characteristic was calculated. A study was also conducted in the case where the coupling determines the elastic characteristics of the Duffing type "soft" and "hard" type.
Keywords: *elastic coupling; mechanical feedback; elastic characteristic; oscillation process; rotational mass; starting torque.*

1. INTRODUCTION

In modern machine-building, elastic couplings with metal elastic elements have become widespread. This is facilitated by the ability of these devices not only to transmit torque, but also to prevent negative oscillations in the technical system. This is achieved by introducing into the design an elastic coupling of mechanical feedback, which provides a wide range of elastic characteristics, including also nonlinear. Studies in this direction have shown that the nonlinearity of the elastic characteristics of one of the components of the machine aggregate can significantly change the nature of the oscillating processes, which occur.

2. PROBLEM STATEMENT

Studies using mathematical model proved that elastic couplings with a nonlinear elastic characteristic show the most positive results [1]. However, already existing elastic couplings do not fully meet the stated requirements due to their narrow working range [2-4]. Up to now created potential designs of elastic couplings that implement a nonlinear elastic characteristic are not widely used due to the small number of their actual mechanical constructions.

3. ANALYSIS OF RECENT RESEARCH AND PUBLICATIONS

At this stage, most of drives use asynchronous motors. The features of their operation, specifically the startup of the engine, cause the considerable oscillatory load on the drive, this is due to the large and short-term starting torque. Because of this, there is a significant number of works devoted to oscillating starting torque. Because of this, there is a significant number of works devoted to oscillating processes in technical systems [5-8]. A mathematical modeling of the start of an asynchronous electric motor was carried out by using software packages [9,10]. Developed the promising designs of nonlinear elastic couplings, which reduce the load on the drive and prevent negative oscillations [11]. The following studies show the feasibility of using elastic couplings with nonlinear mechanical feedback.

4. OBJECTIVES AND PROBLEMS OF RESEARCH

Mathematical modeling of oscillatory process of transmission starting of a machine assembly with an asynchronous electric motor, which consist of an elastic coupling with nonlinear mechanical feedback and studying the effect of elastic characteristics on the magnitude of the amplitude, frequency of the oscillatory process and its time.

5. RESULTS OF THE STUDY

The chosen aim of the research is based on the fact that the results of the researches carried out in the field of nonlinear oscillation mechanics indicate that the nonlinearity of the elastic characteristics of one of the components of the machine assembly can significantly change the nature of oscillatory processes.

In the given research area it is believed that the starting torque M_{start} of the asynchronous motor shaft is a torque that advances on the shaft of an asynchronous electric motor under the following conditions: the speed of rotation is equal to 0, the current has a constant value, the electric motor windings are connected to rated supply frequency and voltage, the winding connection corresponds to the rated operating mode of the electric motor.

In mathematical modeling of the oscillatory processes of the machine assembly, the starting torque $M_s(t)$ is modeled by a function of time characterized by two time intervals: the build-up time t_1 to the maximum value and the time of decrease to the rated value t_2. In order to calculate the maximum starting torque the following expression is used

$$M_{\max} = M_r \cdot k_{tr} , \qquad (1)$$

where M_r – rating moment on the electric motor shaft; k_{tr} – starting torque ratio. The value of this parameter varies within 1,5…6 for different types of engines and loads.

Duration of the starting torque is determined experimentally, depending on the type of engine and the type of its load. Usually the value of this parameter varies within 0,5...1,6 s.

In order to achieve this goal a two-mass rotatory mechanical system (J_1 – main rotating mass, subject to protection against the negative demonstration of the starting torque), which includes the proposed passive elastic coupling with a nonlinear mechanical linkage (J_2 – the second rotating mass, which is the mass of the coupling) should be subject to mathematical modeling. In this case the system of differential equations has the following form

$$\begin{cases} J_1\ddot{\varphi}_1 + b_1\dot{\varphi}_1 - b_2(\dot{\varphi}_2 - \dot{\varphi}_1) + c_1\varphi_1 - c_2(\varphi_2 - \varphi_1) = 0 \\ J_2\ddot{\varphi}_2 + b_2(\dot{\varphi}_2 - \dot{\varphi}_1) + c_2(\varphi_2 - \varphi_1) = M_s(t) \end{cases}. \quad (2)$$

However, the rotating mass of the J_2coupling in several cases is less than the rotating mass of J_1 transmission objects (J_2 >> J_1) and the stiffness of the shaft sections, which determines the torsion angle φ_1, is several times greater than the stiffness of the elastic coupling, which determines the torsion angle φ_1(φ_2>> φ_1).

Taking this into account it is advisable to carry out mathematical studies of the process of transmission starting of a machine assembly with an asynchronous electric motor, which includes the proposed elastic coupling, using a mathematical model of a single-mass rotatory system. In this case, the model treats the rotating mass J_1 as an object to be protected from the negative demonstration of the starting torque, and the elastic coupling is considered as an elastic linkage between it and the engine. Then the corresponding differential equation will have the following form

$$J\ddot{\varphi} + M_{el}(\varphi) + M(\dot{\varphi}) = M_s(t)\,, \quad (3)$$

where J – moment of inertia of the rotating mass; $M_{el}(\varphi)$ – elastic characteristic, which depends on the stiffness of elastic elements applied in the coupling; $M(\dot{\varphi})$ – moment of dissipation, which determines the irreversible energy dissipation; $\dot{\varphi}$ and $\ddot{\varphi}$ – corresponding derivatives of the angular displacement in time t.

Initial conditions are as follows

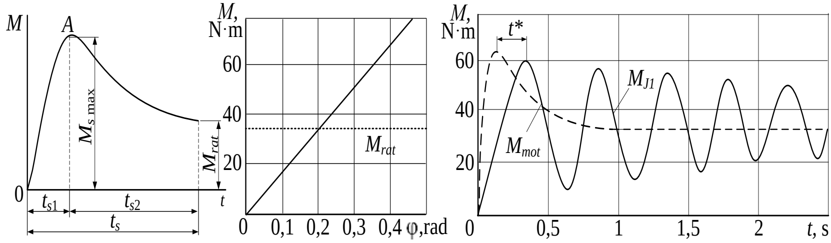
$$\varphi(0) = 0\,,\ \dot{\varphi}(0) = 0\,,\ M_s(0) = 0\,. \quad (4)$$

On the basis of the equation (3) mathematical modeling of the oscillatory processes of transmission starting of a machine assembly with an asynchronous electric motor AIR112MV6 with the following characteristics was carried out: P = 4 kW, n = 1000 rpm; rating moment M_{rat} = 34,5 N□m; starting torque ratio k_{tr} = 1,8; time of the starting torque t_s = 0,8 s.

$$
\begin{aligned}
&t_{s1} = 0\ldots0{,}26\text{s}, && M_{mot} = M_{s1}(t) = 162299t^2 + 6312{,}1t + 3{,}4857; \\
&t_{s2} = 0{,}26\ldots0{,}8\text{s}, && M_{mot} = M_{s2}(t) = 16229t^2 - 4512{,}1t - 2{,}5734; \qquad (5) \\
&t_s > 0{,}8\text{s} && M_{mot} = M_r = 34{,}5\text{N}\cdot\text{m}
\end{aligned}
$$

In calculations the starting torque is presented in the form of two non-linear sections associated with time and has a maximum $M_{s\max}$(0,026)=61 N□m (fig. 1, a).

For the possibility of conducting a comparative analysis in order to determine the appropriate efficiency ratios the calculation of the accepted conditions of the system, which containsan elastic coupling with a linear elastic characteristic, was carried out (fig. 1, b). Using the Maple 18 mathematical package, where the corresponding function implements the Runge-Kutta method, the solution of equation (3) was carried out in numerical form taking into account the initial conditions (4) and external load (5), which made it possible to state the following. Emerging at the process of starting a transmission of a machine assembly with an asynchronous electric motor, the oscillatory process is fading and low frequency with a constant frequency T = 2 Hz (fig. 1, c). Oscillatory processes with the frequency of the first frequency octave, that is T = 2, 4, 8, 16, 31,5 and 63 Hz, refer to the low-frequency oscillatory process. The response of the system to external disturbance in the form of M_{J1} = 59,3 N□m occurs with the delayafter the appearance of the maximum external load equal to t^* = 0,18 s, which is due to the presence of an elastic linkage. Oscillation decay time under condition of $M_{J1} = M_r$ equals t = 6,7s.



a) b) c)

Figure 1 – Modelling of oscillatory processes of transmission starting of a machine assembly: starting torque (a); linear elastic characteristic (b); oscillatory process of transmission starting of a machine assembly with a couplingthat possesses linear elastic characteristic

The coefficient that determines the efficiency of using an elastic coupling with a linear elastic characteristic is the coefficient of vibration isolation

$$k_R = \frac{M_0}{A_0}\ , \qquad (6)$$

where M_0 – amplitude of the moment behind the coupling; A_0 – amplitude of the moment of disturbance.

In this case the coefficient of vibration isolation is

$$k_R = \frac{M_0}{A_0} = \frac{59{,}3}{61} = 0{,}97\ . \qquad (7)$$

Numerical solution of the equation (3), taking into account the general parameters of the system, the initial conditions (4) and the external load (5), is carried out in cases where the elastic characteristic of the coupling is nonlinear.

In the first case the coupling determined an elastic characteristic of a "soft" Duffing type. The value of the elastic torque at a certain nominal torsion angle of half-couplings φ = 0,2 rad was equal to the value of the elastic torque of the previously considered linear characteristic M = 34,5 N□m (fig. 2, a).

Emerging at the start of the transmission of the machine assembly with an asynchronous electric motor, the oscillatory process is fading and low frequency with frequency T, which increases over time (fig. 2, b).

The response of the system to external disturbance in the form of M_{J1} = 57,23 N□m occurs with the delay after the appearance of the maximum external load equal to t^* = 0,38 s, which is defined by the value of the elastic torque, that is less than the similar one in the linear system, and lays in the range of

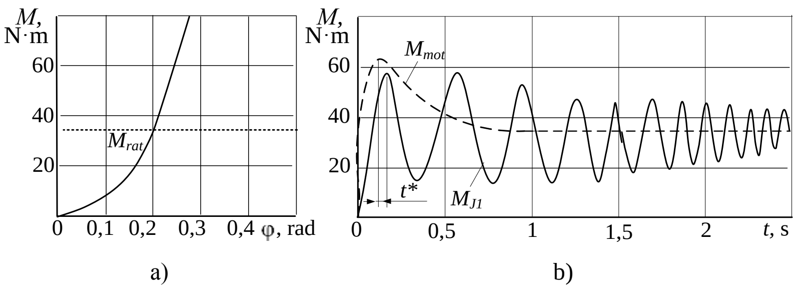


Figure 2 – Oscillatory process of asynchronous motor starting: elastic characteristic of a “soft” Duffing type coupling (a); oscillatory process (b)

the torsion angle of the half-couplings equal to φ = 0,2…0,6 rad. Oscillation decay time, which is determined by $M_{J1} = M_r$, equals $t^* = 3{,}8$ s. The coefficient of vibration isolation k_R in this case is

$$k_R = \frac{M_0}{A_0} = \frac{57{,}34}{61} = 0{,}94\,. \tag{8}$$

In the second case the coupling determined an elastic characteristic of a "hard" Duffing type. The value of the elastic torqueat a certain nominal torsion angle of half-couplings φ = 0,2 rad was equal to the value of the elastic torque of the previously considered linear characteristic M = 34,5 N□m (fig. 3,a).

Emerging at the start of the transmission of a machine assembly with an asynchronous electric motor, the oscillatory process is fading and low frequency with a frequency T decreasing over time (fig. 3, b). The response of the system to external disturbance in the form of M_{J1} = 59,16 N□m occurs with the delay after the appearance of the maximum external load equal to t^* = 0,16 s, which is defined by the value of the elastic torque, that is higher than the similar one in the linear system, and lays in the range of the torsion angle of the half-couplings equal to φ = 0,2…0,6 rad. The decay time of the oscillatory process is determined by $M_{J1} = M_r$ and equals t = 3,4 s. The coefficient of vibration isolation k_r in this case is

$$k_R = \frac{M_0}{A_0} = \frac{59{,}13}{61} = 0{,}96\,. \tag{9}$$

Results of mechanical studies conducted to optimize the oscillatory process during the starting of transmissions with an asynchronous motor show that the use of nonlinear couplings with elastic characteristics of a "hard" Duffing type can reduce the time of the oscillatory process, however itdetermines the transmission load close to the starting torque. Application of nonlinear couplings with elastic

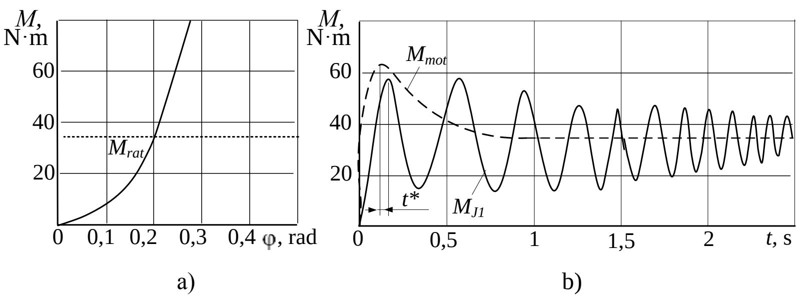


Figure 3 – Oscillatory process of asynchronous motor starting: elastic characteristic of a “hard” Duffing type coupling (a); oscillatory process (b)

characteristics of a "soft" Duffing type allows slight reduction of the transmission load, at the same time it lengthens the time of the oscillatory process. Taking this into account it is proposed to use nonlinear couplings with a combined characteristic in order to solve such a problem. The basis for such a proposal is the results of research done by professor G.V. Arkhangelskiy [12]. It has been established that optimization of the oscillatory process occurring at the start of transmission with an asynchronous motor can be obtained by applying a nonlinear elastic coupling in the transmission, which implements a combined characteristic with two sections, determined by the value of the rating rotary moment. The first section (M = 0…M_r) must correspond the elastic characteristic of the "soft" Duffing type and the second section (M = M_r…1,3M_s) must relate with the elastic characteristic of the "hard" Duffing type. The researcher has proposed a specialized design of an elastic coupling that implements a similar characteristic, but because of structural constraints its elastic characteristic corresponds to the target characteristic with a compliance coefficient equal to k_c = 0,89 and is fragmentarily linear (line 1, see fig. 4, a).

From this perspective the calculations of the oscillatory process during the start of the transmission with the asynchronous motor, while applying the proposed coupling both with mentioned above elastic characteristic (combined, type 1) and with the synthesized target characteristic with the coefficient of compliance k_c = 0,99 (hereafter combined, type 2) have been carried out. The synthesized elastic characteristic consists of the corresponding nonlinear sections that share borders at a certain value of the elastic torque and determine the rating rotary moment of half-couplings φ = 0,2 rad. (curve 2, fig. 4, a). Emerging at the start of the transmission of a machine assembly with an asynchronous motor in two calculation cases the oscillatory process is fading and low frequency with the frequency T, which varies over time (fig. 4, b). The response of the system to external disturbance in the form of $M_{J1(t1)}$ = 56,26 N□m occurs with the delay after the appearance of the maximum external load in the first case equal to $t^*_{(t1)}$ = 0,18 s and in the second $M_{J1(t2)}$ = 56,26 N□m with the delay equal to $t^*_{(t1)}$ = 0,21 s. Thus it is established that the average value of the elastic torque lays in the range of the torsion angle of the half-coupling

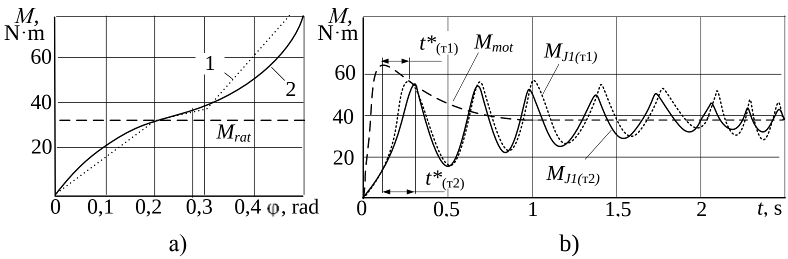


Figure 4 – Oscillatory process of asynchronous motor starting: combined elastic charactericts (a); oscillatory process (b)

φ = 0,2…0,6 rad, which is higher than in the linear system and less than in a system with a "hard" Duffing type characteristic. The decay time of the oscillatory process is determined by $M_{J1} = M_r$ and in the first case $t_{(t1)}$ = 3,18 s, while in the second case $t_{(t2)}$ = 2,8 s, being the smallest indicators in the performed calculations. This is due to the fact that at high amplitudes of oscillations elastic characteristics cause an increase in their frequency. This, in turn, indicates the presence of high velocities and the greater effect of dissipative forces than in the previously considered variants. In this case the coefficient of vibration isolation k_R for the first calculation is as follows

$$k_R = \frac{M_0}{A_0} = \frac{56{,}79}{61} = 0{,}931\,, \qquad (10)$$

and for the second calculation

Table 1 – Coefficients of vibration isolation k_R and oscillation decay time at the start of the transmission with asynchronous motor with an elastic coupling

Type of elastic characteristic of coupling	Coefficient of vibration isolation k_R	Oscillation decay time *t*, (s)
Linear	0,98	6,7
"Soft" Duffing type	0,94	8,3
"Hard" Duffing type	0,96	3,4
Combined, type 1	0,93	3,2
Combined, type 2	0,92	2,8

$$k_R = \frac{M_0}{A_0} = \frac{56{,}13}{61} = 0{,}92\,. \qquad (11)$$

The results of the conducted analytical studies are presented in the table 1.

6. CONCLUSIONS

Implementation of elastic characteristics of the "soft" Duffing type of the coupling in comparison with the case of implementation of a linear elastic characteristic of the coupling enables reduction of negative demonstrations of oscillations by 3...4%, however it leads to an increase in duration of oscillatory process 1,5...2 times. Implementation of elastic characteristics of the "hard" Duffing type of the coupling in comparison with the case of implementation of a linear elastic characteristic of the couplingallows to reduce the negative demonstrations of oscillations by 2...3% and leads to a decrease in duration of oscillatory process 1,5...2 times.

Results of research of prof. G.V. Arkhangelskiy conserning optimization of oscillatory process in case of start of transmission of a machine assembly with an asynchronous motor using elastic coupling with a combined nonlinear elastic characteristic have been confirmed. Mathematical modeling of the system starting with the proposed coupling structure, which implements the target characteristic in the form of a fragmentarily linear characteristic with a compliance coefficient k_c = 0,89, resulted in a decrease of negative demonstration of oscillations by 5...6% and reduction of the time of oscillatory process 1,5...2,5 times, comparing with the case of realization of a linear elastic characteristic by a coupling.

Mathematical modeling of the system starting using an elastic coupling with a mechanical feedback that implements a nonlinear target combined characteristic with a compliance coefficient k_c=0,98 caused a decrease of negative demonstration of oscillations by 7...10 %, and reduction of the time of oscillatory process 2,8...3 times, compared with the case of realization of a linear elastic characteristic by a coupling.

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МАТЕМАТИЧНЕ МОДЕЛЮВАННЯ ПУСКУ ТРАНСМІСІЇ З АСИНХРОННИМ ДВИГУНОМ

Анотація. *Найскладнішим моментом в роботі приводу з асинхронним електродвигуном є запуск. І чим потужніший привід - тим цей запуск складніший. Це пов'язано з певними особливостями асинхронних двигунів: обмеженого пускового моменту і пускових кидків струму ланцюга статора двигуна. В роботі проведено математичне моделювання коливного процесу пуску приводу з асинхронним електродвигуном, до складу якої входить пружна муфта з нелінійним механічним зворотним зв'язком. Досліджувався вплив виду пружних характеристик муфти на величини амплітуди та частоти коливного процесу та його час. Для досліджень використана модель одномасової обертальної системи. За методом Рунге-Кутта проведено дослідження коливних процесів пуску трансмісії машинного агрегату з асинхронним електродвигуном. Для визначення коефіцієнту ефективності віброізоляції виконано розрахунок системи, що має у своєму складі пружну муфту з лінійною пружною характеристикою. Також проведено дослідження у випадку, коли муфта обумовлює пружні характеристики Дуфінговського типу "м'якого" і "жорсткого" виду. Результати розрахунків показують, що доцільним є використання нелінійної муфти з комбінованою характеристикою. На підставі цього проведено синтез цільової пружної характеристики і дослідження коливного процесу при застосуванні запропонованої пружної муфти. Результати дослідження показують, що використання муфт з пружними характеристиками "жорсткого" типу дозволяє скоротити час коливальних процесів, однак викликати навантажувальні передачі, близькі до пускового моменту, і з пружними характеристиками "м'якого" типу, дозволяють трохи зменшити навантаження на передачу, проте подовжують час коливального процесу.*

Ключові слова: *пружна муфта, механічний зворотний зв'язок, пружна характеристика, процес коливань, обертальна маса, пусковий момент.*