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OPTIMISATION OF PURCHASING STRATEGY OF TOOLS AND COMPONENTS BASED ON EXCHANGE CURVE THEORY

Abstract. *The optimal inventory control of tools and components in manufacturing systems plays an important role in the success of sustainable operation from financial and ecologic point of view. This study discusses an inventory control method, which is based on the transformation of investment of inventory into annual order cost or vice versa. The study presents the mathematical model of the exchange curve model for tools and components in the case of economic order quantity inventory strategy. The described methodology makes it possible to optimise available purchasing strategies for tools and components. The approach was tested with a scenario analysis, where different parameters of the purchasing process including inventory related constraints were taken into consideration. The computational results validated the exchange curve based inventory control methodology and showed that the inventory strategy can be improved with cost transformation Practical implications of the proposed model and method regard the possibility of finding optimal inventory policies that can affect the operation costs of manufacturing systems.*
Keywords: *purchasing logistics; optimisation; inventory control; exchange curve.*

1. INTRODUCTION

Today, in order to increase the efficiency of production systems and meet the dynamically changing demands of customers, manufacturing companies are doing their utmost to optimise not only their production systems, but also the related service processes, especially focusing on logistics. This means that the optimisation of procurement, distribution and recycling logistics activities is becoming increasingly important for sustainable production systems. In the field of production logistics, strategic issues can be defined in two cases: design of a new production logistics systems or improve an existing production logistics systems. Logistics strategies should support the following production logistics objectives: increasing capacity utilisation of production and logistics resources, reducing lead times, reducing production process inventories (work in process) without increasing supply risk, reducing the operation costs of the technological and logistical process, increasing flexibility a reliability, increasing the transparency of the technological process, reducing the environmental impact and integration of the production logistics process into the overall company logistics system [1].

This paper studies the optimisation potentials of purchasing policies of tools and components in a manufacturing system, while the economic order quantity is taken into consideration. As the literature review section will show, modern optimisation algorithms play an important role in the design and operation of logistics systems [2] and the majority of the articles in the field of purchasing policy optimisation are focusing on conventional manufacturing environment and only a few of them describes the purchasing policy optimization in a cyber-physical system, where available real time status information based on digital twin technology makes it possible to define a dynamic purchasing strategy optimisation of tools and components. The article is organized as follows. Section 2 presents a systematic literature review, which summarizes the research background of purchasing policy optimization based on exchange curve. Section 3 describes the mathematical model of exchange curve-based purchasing policy optimisation including three different scenarios depending on the constraints. Section 4 demonstrates the scenario analysis, which validates the model. Conclusions, future research directions and managerial impacts are discussed in Section 5.

2. LITERATURE REVIEW

As previous studies show, the optimisation of the inventory level and the costs of inventory management has a great impact on the competitiveness of manufacturing processes. A wide range of objective functions and constraints can be taken into consideration regarding purchasing policy optimisation: purchasing and stock holding costs; financial, technological and logistics consequence of shortages; uncertainties of the manufacturing and service environment [3-5] or the warehousing system [6]. The technologies of the fourth industrial revolution and the transformation of conventional manufacturing and service systems into cyber-physical systems lead to a more complex supply chain structure, where the four supply chain levers have to be taken into consideration as an integrated, interconnected or hyperconnected system, where the key performance indicators of the supply chain subsystems (procurement, warehousing, fulfilment and transportation) have a great impact on each other [7]. Statistical survey shows, that the application of Industry 4.0 technologies leads to a cost savings of about 10% regarding administrative costs for each purchasing [8]. Industry 4.0 technologies are changing the role of purchasers, because the co-creation of specifications, automatized prequalification and negotiations lead to the improvement of procurement processes [9,10], especially from reduced uncertainties resulted by the application of digital twin technologies point of view. The big data oriented smart tool condition monitoring makes it possible to evaluate the present status of the tools and make prediction including lifetime expectancies, which can support the optimisation of tool procurement processes [11].

About 60% of the articles were published in the last five years. This result indicates the scientific potential of the research of purchasing policy optimisation in Industry 4.0 era. The articles that addressed the development of purchasing policy optimisation are focusing on different fields of procurement and buying aspects of supply chain solutions in the field of manufacturing, but only a few of them focuses on the potentials of Industry 4.0 technologies. As a consequence, the main contribution of this article is the exchange curve-based mathematical model of purchasing policy optimisation and the description of computational results validating the described model.

3. TOOL AND COMPONENT MANAGEMENT IN CYBER-PHYSICAL MANUFACTURING

However, there are different ways to optimise conventional manufacturing systems, but Industry 4.0 technologies offer new opportunities and potentials to improve the optimisation, especially focusing on the real time data-based design and operation based on smart sensors and digital twin solutions. In cyber-physical manufacturing systems, it is possible to use the real time status information to optimize the manufacturing related operations including purchasing, distribution and in-process recycling and reuse. Figure 1 shows a possible solution of the optimisation of purchasing strategies of tools and components of a cyber-physical manufacturing system. The proposed model includes the following main phases:

1. Smart micro sensors can be mounted into intelligent tools, and these sensors can collect real time status information of the tool (temperature, deformation, tension, etc.).
2. The collected information can be uploaded indirectly to a database, or directly to a digital twin solution.
3. The digital twin solution can build real time models of the analysed tool, or tool-machine, or tool-machine-product system. This real time model can be uploaded into a discrete event simulation software, where the model represents the present status of the system.
4. Using discrete event simulation, it is possible to analyse the modelled system and make predictions.
5. The tool inventory is a basic information for the prediction, especially if this information focuses on both quality and quantity of available tools.
6. The enterprise resource planning includes a wide range of modules, from these the production planning and scheduling module is responsible for the design, operation, supervision and controlling of the manufacturing system.
7. The production planning and scheduling module determines the master production schedule, which is the most important input parameter for the purchasing policy.
8. Using the master production schedule, it is possible to define resource demands (machine tools, tools, components, human resources, logistics resources, packaging).

9. Based on the status information of inventory of tools and components and the prediction information, the future potential status of the system can be predicted, which is the basic information for the optimisation of the purchasing policy.

10. Based on the prediction regarding tools, it is possible to make predictions according the whole analysed systems including tools and components.

11. The master production schedule is an important input parameter of the purchasing policy optimisation.

12. The optimised purchasing policy has a great impact on the component inventory, because the resulted changes in annual order cost and annual investment in inventories of components lead to new parameters of the purchasing process, which influences in this way the component inventory.

13. The optimised purchasing policy has a great impact on the tool inventory, because the resulted changes in annual order cost and annual investment in inventories of tools lead to new parameters of the purchasing process, which influences in this way the tool inventory.

14. Predictions and lifetime expectancies of tool and products is an important input parameter for the exchange curve-based purchasing policy optimisation.

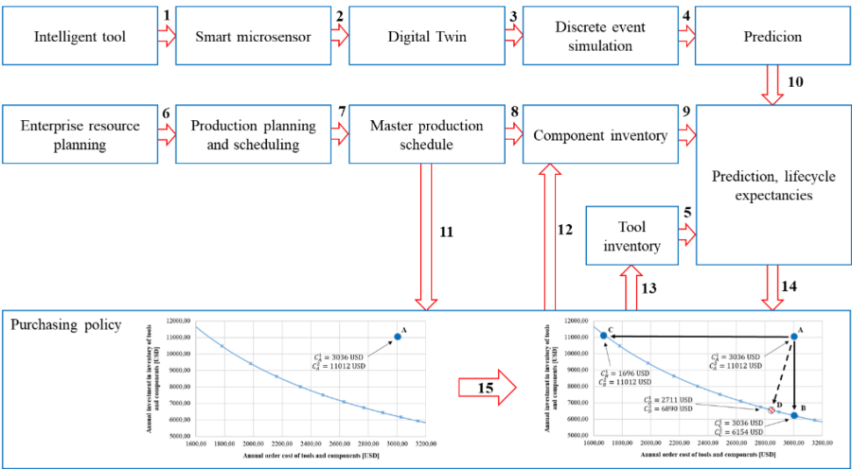

Figure 1 – Digital twin-based optimization of purchasing policy of tools and components

4. MATHEMATICAL MODEL

To optimise the operation of production systems, it is essential to design related logistics processes in a cost-effective way. Since one of the key elements of efficiency in production systems is the availability of tools and parts, it is of great importance to define an optimal inventory strategy. Inventory management processes have great importance in company's logistics system and have a major impact on its economic operation. The aim of inventory management is to smooth out the fluctuations in quantity resulting from imbalances between input and output flows in the material system or from external environmental disturbances.

The purchasing policy of tools and components of manufacturing systems can be optimised in many ways, depending on the characteristics of goods. If the purchasing policy is based on economic order quantity methodology, then in the mathematical model of the exchange curve based inventory control the average inventory investment can be defined depending on the specific purchasing cost of tools and components:

$$C^1 = \sum_{l=1}^{p} \frac{q_l}{2} \cdot \alpha_l. \tag{1}$$

where α_l is the specific purchasing cost of tools and components and q_l is the economic order quantity, which can be calculated using the following equation:

$$q_l = \sqrt{\frac{2 \cdot AD_l \cdot \alpha_l}{\beta_l \cdot \gamma}}, \tag{2}$$

where AD_l is the annual demand of tools and components, α_l is the specific order cost of tools and components, β_l specific purchasing cost of tools and components, γ is the specific warehousing cost of tools and components.

Using the economic order quantity for all tools and components, the average inventory investment can be calculated:

$$C^1 = \frac{1}{2} \cdot \sum_{l=1}^{p} \beta_l \cdot \sqrt{\frac{2 \cdot AD_l \cdot \alpha_l}{\beta_l \cdot \gamma}} = \frac{1}{\sqrt{2 \cdot \gamma}} \cdot \sum_{l=1}^{p} \sqrt{AD_l \cdot \alpha_l \cdot \beta_l}. \tag{3}$$

The second part of the cost function is based on the annual order cost of tools and components, which can be calculated in the following way:

$$C^2 = \sum_{l=1}^{p} \frac{AD_l \cdot \alpha_l}{q_l}. \tag{4}$$

Using the economic order quantity for all tools and components, the annual order cost can be calculated in the following way:

$$C^2 = \sum_{l=1}^{p} AD_l \cdot \alpha_l \cdot \sqrt{\frac{\beta_l \cdot \gamma}{2 \cdot AD_l \cdot \alpha_l}} = \sqrt{\frac{\gamma}{2}} \cdot \sum_{l=1}^{p} \sqrt{AD_l \cdot \alpha_l \cdot \beta_l}. \tag{5}$$

Depending on the value of the average inventory investment and the annual order cost, it is possible to draw a curve between these costs. This $C^1 - C^2$ curve makes it possible to analyse and optimise the present inventory policy of tools and components depending on the position of the present inventory policy related to the $C^1 - C^2$ curve (see Figure 2).

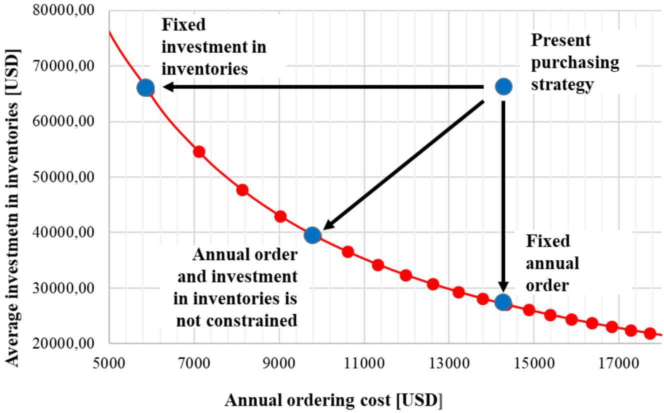

Figure 2 – Potential optimization directions of the purchasing policy depending on the position of the present situation related to the exchange curve

Depending on the constraints defined by the purchasing policy based on economic order quantity, we can define three different types of optimisation of purchasing policy:

- In the first case, the purchasing policy defines the number of annual orders regarding tools and components as a constant value (annual order constraint). Depending on the parameters of the purchasing policy, we can minimize the inventory costs, while it is not allowed to change the number of annual orders which lead to a constant annual order cost. In this case, the optimization of the purchasing policy means the transformation of investments in inventories of tools and components to annual order costs.
- In the second case, the purchasing policy defines the investment in inventories of tools and components as a constant value (investment in inventory constraint). Depending on the parameters of purchasing policy, we can minimize the annual order costs, while it is not allowed to change the investments in inventory of tools and components. In this case, the optimization of the purchasing policy means the transformation of annual order costs to investments in inventories of tools and components.
- In the third case, neither the annual orders nor the investment in inventories of tools and components are constrained by the purchasing policy

based on economic order quantity model. In this case, we are talking about a mixed transformation, where we have to find the nearest point on the $C^1 - C^2$ curve, which means, the it is possible that both parts of the purchasing cost will be changed. In the case of this purchasing optimisation we can minimize the change in the parameters related to the present purchasing strategy.

This $C^1 - C^2$ plot is a rectangular hyperbola, which is named as an exchange curve:

$$C^1 \cdot C^2 = \frac{1}{2} \cdot \left\{ \sum_{l=1}^{p} \beta_l \cdot \sqrt{AD_l \cdot \alpha_l \cdot \beta_l} \right\}^2 \tag{6}$$

Within the frame of the next chapter some scenarios and numerical exampled demonstrate the usability and efficiency of the exchange curve-based purchasing policy optimisation in the case of tools and components in a manufacturing system.

5. NUMERICAL EXAMPLES

In the case of conventional manufacturing systems, where no real time status information is available it is almost impossible to perform a real time optimization a smoothing of purchasing policy. In cyber-physical systems, where the digital twin of the manufacturing system collects status information from the resources and processes of the manufacturing system through smart sensors, it is possible to make real time decisions and optimize the purchasing strategy regarding tools and components. In the case of intelligent tools, the inbuilt micro sensors can collect status information, and based on this information it is possible to predict the future status and life expectancy and the purchasing policy can be permanently changed.

In this scenario, the purchasing policy of tools and components in a cyber-physical manufacturing system is analysed. The input parameters of the purchasing optimization problem are given in Table 1, including the annual demand of tools and components depending on the master production plan based on ERP, the specific order cost and purchasing cost of tools and components (see Table 1). The company orders the required tools and components monthly (12 times yearly) and the annual order cost of this solution is 3036 USD, while the average inventory investment is 11012 USD.

Table 1 – Input parameters of the purchasing policy optimisation problem

ID	X1201	X1202	X1203	X1204	X1205	X1206	X1207
α_l [USD]	320	200	80	90	50	90	120
AD_l [pcs]	450	260	320	215	35	120	90
β_l [USD]	10	30	50	40	22	34	67

Depending on the specific warehousing cost of tools and components (γ), it is possible to compute both parts of the cost function regarding purchasing of tools and components. As Figure 3 demonstrates, regarding to the methodology of the economic order quantity, the specific warehousing cost has the opposite impact on the annual order cost and on the annual investment in inventory. The increased specific warehousing cost lead to increased annual order cost, while the increased specific warehousing cost lead to decreased annual investment in inventory of tools and components.

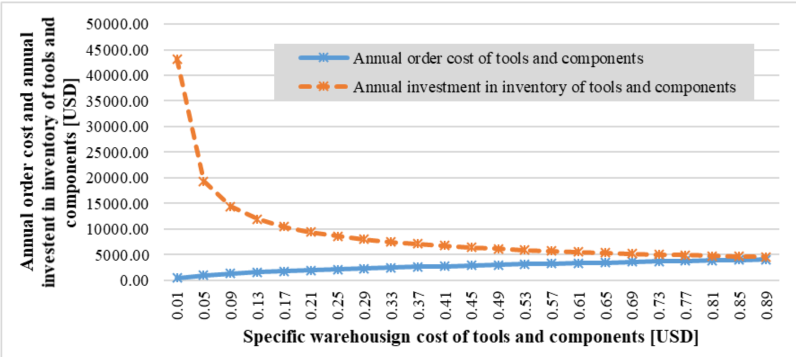

Figure 3 – Annual order cost and annual investment of inventory of tools and components depending on the specific warehousing cost

As Figure 4 demonstrates, in the case of this scenario there are three different ways to optimize the present purchasing strategy of tools and components (see point A in Figure 4). In the case of monthly order of tools and components, the annual order cost is 3036 USD and the annual investment in inventories is 11012 USD. The three potential optimisation directions are the followings:

- We can fix the annual order cost of tools and components and it is possible to find a potential solution on the $C^1 - C^2$ curve with a reduced investment in inventory of tools and components. This solution is represented by point B in Figure 4, where the annual order cost is 3036 USD and the annual investment in inventories is 6154 USD, which results a total annual purchasing cost reduction of 4858 USD.
- We can fix the investment in inventory of tools and components and it is possible to find a potential solution on the $C^1 - C^2$ curve with a reduced annual order cost of tools and components. This solution is represented by point C in Figure 4, where the annual order cost is 1696 USD and the annual investment in inventories is 11012 USD, which results a total annual purchasing cost reduction of 1340 USD.

- In the case of the third potential way, there are no constraints defined for the annual order and investment in inventory, therefore we can find a potential solution on the $C^1 - C^2$ curve to minimize the distance between point representing the present solution and point representing the new solution. A new procedure to find the minimum distance from the operating point representing the present purchasing strategy makes it possible to find this solution [12], which represents a potential solution with minor changes in the purchasing strategy. Using the suggested methodology, we can define this minimal distance point (see point D in Figure 4), where the annual order cost is 2711 USD and the annual investment in inventories is 6890 USD, which results a total annual purchasing cost reduction of 4447 USD.

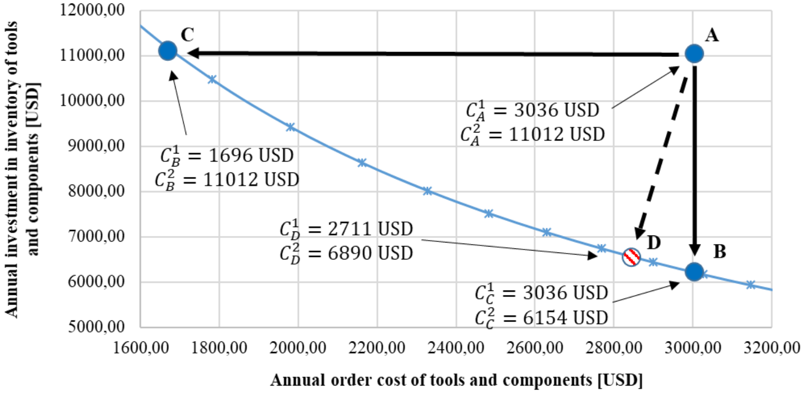

Figure 4 – Potential optimization directions of the purchasing policy

The above mentioned purchasing policy optimisation is a macro-level optimisation, because this methodology supports the detection of bottlenecks in purchasing strategy regarding annual order cost and annual investment in inventories of tools and components. After finding the bottleneck and changing the better strategy for the purchasing the next important task is to perform a micro-level purchasing policy optimisation, which can be based in this case on the economic order quality methodology, because after finding a better macro-level strategy on the $C^1 - C^2$ curve we can compute the new parameters for the new policy and we can determine the new parameters for the purchasing of tools and components while depending on the manufacturing and logistics environment different other components of the whole manufacturing system must be taken into consideration (triggers in Kanban systems, stock out management, age management and safety stock planning).

6. CONCLUSIONS

The efficiency of production systems is affected by the efficiency of both technological and logistic subsystems. An important factor of the optimisation of production systems is the optimisation of available resources, which is an important task of purchasing logistics, especially for tools and components. Within the frame of this article the author described an exchange curve-based purchasing strategy optimisation approach, which makes it possible to optimise the present purchasing policy depending on the value of annual order cost and annual investment in inventory of tools and components. The proposed model focuses on the potentials of Industry 4.0 technologies and describes a solution including intelligent tools, smart micro sensors, digital twin technology and discrete event simulation. Purchasing policies are extensively discussed in a wide range of research works, but only a few of them focuses on the potentials of real time optimisation using Industry 4.0 technologies. To try to fill this gap, this work has introduced a purchasing policy optimization methodology based on exchange curve theory to analyse the structure of purchasing cost of tools and components and suggest more efficient policies for the analysed purchasing. The described methodology shows that the optimization of purchasing policy in cyber-physical systems has a great impact on the profit of the manufacturing system and digital twin solutions can support the optimization of purchasing policy, because real-time data collection can improve the efficiency of failure data forecast, prediction and status information collection. The most important managerial impact of this methodology is, that the application of the proposed model and method can support managerial decisions regarding buying, purchasing, procurement and warehousing. A further study of the proposed work would be the modelling and optimisation of the fourth optimisation direction, which objective function is the maximisation of the profit between the present purchasing policy and the new purchasing policy regarding the $C^1 - C^2$ rectangular hyperbola.

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ОПТИМІЗАЦІЯ СТРАТЕГІЇ ЗАКУПІВЕЛЬ ІНСТРУМЕНТІВ І КОМПЛЕКТУЮЧИХ НА ОСНОВІ ТЕОРІЇ ОБМІННИХ КРИВИХ

Анотація. *На ефективність виробничих систем впливає ефективність як технологічної, так і логістичної підсистем. Важливим фактором оптимізації виробничих систем є оптимізація наявних ресурсів, що є важливим завданням закупівельної логістики, особливо інструментів та комплектуючих. У рамках цієї статті автор описав підхід до оптимізації стратегії закупівель на основі кривої обміну, що дозволяє оптимізувати існуючу політику закупівель залежно від величини річної вартості замовлення та щорічних інвестицій у запаси інструментів та комплектуючих. Пропонована модель фокусується на можливостях технологій Індустрії 4.0 та описує рішення, що включає інтелектуальні інструменти, інтелектуальні мікродатчики, технологію цифрових двійників та моделювання дискретних подій. Політика закупівель широко обговорюється у великій кількості досліджень, але лише деякі з них присвячені можливостям оптимізації в реальному часі з використанням технологій Індустрії 4.0. Щоб спробувати заповнити цю прогалину, у цій роботі була представлена методологія оптимізації політики закупівель, що базується на теорії кривої обміну, для аналізу структури закупівельної вартості інструментів і компонентів та пропозиції більш ефективних політик для аналізованих закупівель.Описана методологія показує, що оптимізація політики закупівель у кіберфізичних системах дуже впливає на прибуток виробничої системи, а рішення цифрових двійників можуть сприяти оптимізації політики закупівель, оскільки збір даних в режимі реального часу може підвищити ефективність прогнозування даних щодо відмов, прогнозування та збору інформації про реальний стан. Найбільш важливим управлінським ефектом цієї методології є те, що застосування запропонованої моделі та методу може підтримувати управлінські рішення щодо закупівель, постачання та складування. Подальшим дослідженням запропонованої роботи буде моделювання та оптимізація четвертого напряму оптимізації, цільовою функцією якого є максимізація прибутку між існуючою політикою закупівель та новою політикою закупівель щодо прямокутної гіперболи* C^1-C^2.

Ключові слова: *закупівельна логістика; оптимізація; контроль запасів; крива обміну.*