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THEORETICAL VALUE AND EXPERIMENTAL STUDY OF ARITHMETIC MEAN DEVIATION IN ROTATIONAL TURNING

Abstract. *The calculation method of the Arithmetic Mean Deviation (R_a) is presented for rotational turning. The necessary equations for the calculation of R_a are given beginning from the equation of the cut surface theoretical profile, and we determine the theoretical values in the studied range of the technological parameters. Cutting experiments with these data were performed and the roughness values of the machined surfaces were measured. Then we carried out a comparative analysis of the measured and the calculated values of the arithmetic mean deviation.*

Keywords: *arithmetic mean deviation; rotational turning; theoretical and experimental study.*

1. INTRODUCTION

In the production of surfaces and parts, which meet the prescribed roughness requirements, several types of procedures with different kinematic and/or geometric relation are applied in finishing operations [1,2]. The variety of kinematics comes from the different translational or rotational movements of the workpiece and tool [3,4], while the diversity of geometry is given by the number of edges, the design and position of the cutting edge (for example: linear or helical), the shape of the removed chip (constant or varying) and the characteristic of material removal (continuous or intermittent) [5,6]. The selection of the machining procedure is determined by the prescribed accuracy and roughness (quality) requirements among others [7,8], furthermore the more productive one should be chosen for several optional procedures [5,6,7]. However, the increasing costs with the increase in tool wear should be considered as well [9,10]. Longitudinal turning is most widespread in the machining of cylindrical surfaces [11,12].

We study rotational turning, where a helical edged cutting tool with slow rotations removes the allowance from a fast-rotating workpiece with constant chip cross-section and continuous material removal [13,14]. In the studied procedure the machining is done by one cutting edge with constant circular feed and depth of cut. The non-linear (helical) cutting edge and the applied circular feed alter the characteristics of chip removal from the longitudinal cutting; the extent and ratio of cutting force components is different due to the applied edge geometry and kinematics. The contact length between the cutting edge and the workpiece is longer, which is caused by the 0° major cutting edge angle and the inclination angle determined by the helix angle. Therefore, the topography of the machined surface is different. The contact point between the machined surface and the tool moves continuously in one direction along the cutting edge during the cutting.

Hence not just one point of the cutting edge comes into contact with the workpiece during material removal. The usage, load and tool wear will be even along the entire cutting edge. These differences must be taken into consideration in the determination of the process parameters. The effects of the influencing parameters must be known in different machining procedures which can achieve the prescribed roughness values on the part surfaces. The study of these can be done by theoretical (mathematical deduction or modelling [15,16]) and/or experimental analyses, or the comparative analysis of the two [17].

The theoretical values of the roughness parameters give important information on the effect of cutting edge geometry of feed alteration by helping estimate the expected roughness. Therefore, we worked out the calculation method of the Arithmetic Mean Deviation (R_a) for turning with circular feed, and the outcome is compared with experimental results.

2. THEORETICAL VALUE OF ARITHMETIC MEAN DEVIATION

When determining the machined surface roughness, it must be taken into consideration that the theoretically determinable and the practically measurable values are usually different, although a strict correlation can be observed between the two. The theoretical roughness is defined by the position of the cutting edge in the tool reference plane and the feed that is generating the surface periodicity. In rotational turning, the former results from the machined surface radius (r_m), the radius (r_s) and the inclination angle (λ_s) of the helical cutting edge. The periodicity of the surface is given by the tool and workpiece revolutions or angular speed (ω_s and ω_m), the additional axial feed rate ($v_{s,a}$), the inclination angle and the radius of the cutting tool.

In our study, the highlighted section (I-III) of the cut surface must be evaluated for the determination of the arithmetic mean deviation, due to the symmetric nature of the periodically repeating cut surface.

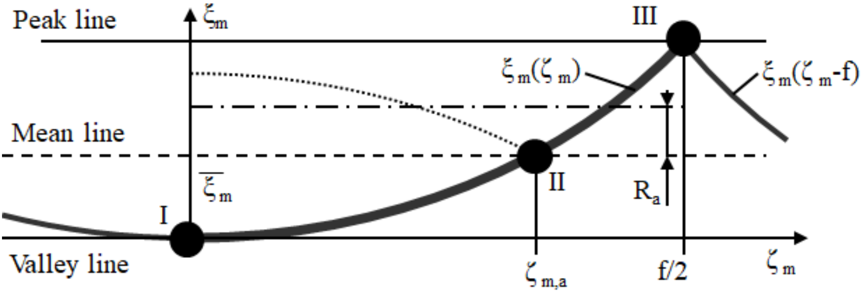

Figure 1 – Arithmetic Mean Deviation (R_a) in the theoretical machined surface

The Arithmetic Mean Deviation can be given by the solution of Equation (1) according to the standard [18] for the theoretical profile (f_a: axial feed, ξ_m: dependant variable, ζ_m: independent variable, $\bar{\xi}m$: value of the mean line).

$$R_a = \frac{1}{f_a}\int_{-f_a/2}^{f_a/2} \left|\xi_m - \bar{\xi}_m\right| d\zeta_m \tag{1}$$

The determination of the cut surface equation is presented in our earlier work [19]; one variable $\xi_m(\zeta_m)$ form can be seen in Equation (2).

$$\xi_m(\zeta_m) = \sqrt{r_s^2 - \left((r_s + r_m)\sin\frac{\zeta_m \omega_m \tan(\lambda_s)}{\omega_m r_m + \omega_s r_s + \frac{v_{s,a}}{\tan(\lambda_s)}}\right)^2} - \\ -(r_s + r_m)\cos\frac{\zeta_m \omega_m \tan(\lambda_s)}{\omega_m r_m + \omega_s r_s + \frac{v_{s,a}}{\tan(\lambda_s)}} \tag{2}$$

The equation of the line parallel to the horizontal axis (mean line) is $\xi_m = \bar{\xi}m$, where the areas below and above the curve are the same. The sought value results from the quotient of the integral on the ζ_m axis of the analysed curve (I-III) and the length of the assessed profile. This results in Equation (3) for the case presented in Figure 1.

It follows from the interpretation above that Equation 1 can be transformed here. Due to the symmetric nature of the studied profile, the lower limit of the integration is 0 instead of $-f_a/2$. The interval can be divided into two sections since the area below (I-II) and above (II-III) the mean line can be calculated separately. The I-III interval is divided into two section by coordinate $\zeta_{m,a}$, where the mean line intersects the analysed curve(point II in Figure 1). This condition can be written as $\xi_m(\zeta_{m,a}) = \xi_m$. Hence the theoretical value of R_a is the solution of Equation (4).

$$\bar{\xi}_m = \frac{2}{f_a}\int_0^{f_a/2} \xi_m(\zeta_m)\, d\zeta_m \tag{3}$$

$$R_a = \frac{2}{f_a}\left[\int_0^{\zeta_{m,a}} (\bar{\xi}_m - \xi_m)\, d\zeta_m + \int_{\zeta_{m,a}}^{f/2} (\xi_m - \bar{\xi}_m)\, d\zeta_m\right] \tag{4}$$

The next step in our study is the determination of the mean line position in the roughness profile of the machined surface. Equation (1) was approximated to perform the designated integration. 6th Taylor polynomial is calculated due to the shape and attributes of the function. The result can be seen in Equation (5). After the evaluation of Equation (3) with the function in Equation (5), the vertical position of the mean line is deduced in Equation (6).

$$\xi_m(\zeta_m) = \frac{\omega_m^2 {\zeta_m}^2 r_m (r_s + r_m)}{2 r_s \left(v_{s,a} + (r_s\omega_s + r_m\omega_m)cot(\lambda_s)\right)^2} \tag{5}$$

$$\bar{\xi}_m = \frac{r_m \pi^2 \left(v_{s,a} + r_s\omega_s cot(\lambda_s)\right)^2 (r_s + r_m)}{6 r_s \left(v_{s,a} + (r_s\omega_s + r_m\omega_m)cot(\lambda_s)\right)^2} \tag{6}$$

The next step is the determination of $\zeta_{m,a}$ from the given condition ($\xi_m(\zeta_{m,a}) = \bar{\xi}m$). After the substition of Equations (5) and (6) into the condition, the sought value is given in Equation (7).

$$\zeta_{m,a} = \frac{\pi\sqrt{3}}{3\omega_m}\left(\frac{r_s\omega_s}{tan\lambda_s} + v_{s,a}\right) \tag{7}$$

The evaluation of the equations leads us to the theoretical value of the Arithmetic Mean Deviation in the form of Equations (8)-(9).

$$R_a = \frac{\frac{2\sqrt{3}}{27}\pi^2\left(r_m + \frac{r_m^2}{r_s}\right)\left[\frac{{r_s}^2\omega_s^2 - v_{s,a}^2}{2}cos^2\lambda_s + r_s\omega_s A + \frac{v_{s,a}^2}{2}\right]}{\frac{(r_s\omega_s + r_m\omega_m)^2 - v_{s,a}^2}{2}cos^2\lambda_s + (r_s\omega_s + r_m\omega_m)A + \frac{v_{s,a}^2}{2}} \tag{8}$$

$$A v_{s,a} sin(\lambda_s) cos(\lambda_s) \tag{9}$$

3. CONDITIONS OF THE CUTTING EXPERIMENTS

The main aim of the cutting experiments was the comparison of the measured and calculated values of R_a. A Perfect-Jet MCV-M8 machining centre was chosen for our studies. The helical edged rotational turning tool was clamped on the machine table parallel with the workpiece placed in the spindle of the machine tool. The rotary feed is caused from the circular interpolation of the tool and the cutting speed is resulted from the rotational movement of the workpiece. The position of the tool and the workpiece can be seen in Figure 2.

The experiments are carried out on heat-treated C45 cylindrical steel workpieces with Ø40 mm diameter and 12 mm length. The material removal is done by a Fraisa P5300682 cutting tool with 30° inclination angle. 200 m/min cutting speed and 0.1 mm depth of cut were adjusted. We analysed the effect of the feed alteration with 7 kinds of values: 0.1 mm/rev., 0.2 mm/rev., 0.4 mm/rev., 0.6 mm/rev., 0.8 mm/rev., 1.0 mm/rev., 1.2 mm/rev. 2D surface roughness was measured by a Mitutoyo SurfTest SJ-301 device on 3 different generatrix of the machined surfaces. The evaluation and cut off lengths were chosen according to the DIN EN ISO 4288 standard.

Figure 2 – Position of the tool and the workpiece

4. EXPERIMENTAL RESULTS AND DISCUSSION

The theoretical and experimental R_a values were evaluated within the studied range. The theoretical values of the Arithmetic Mean Deviation ($R_{a,c}$) were calculated for the different setups using Equation (8). The averages of each of the 3 measured values ($R_{a,m}$) were also determined for the 7 parameter combinations after the cutting experiments. The results of the evaluation are shown in Table 1. Typically, the calculated values of R_a are lower than the measured values.

It can be seen based on the depiction of the theoretical and experimental values in function of the feed (Figure 3) that the roughness alteration caused by the increasing feed is well described by the theoretical curve, the tendency is the same. The coefficient of determination in the studied range is $R^2 = 0.8543$.

Table 1 – Results of the roughness measurement and calculations

f	**a**	**v_c**	**$R_{a,1}$**	**$R_{a,2}$**	**$R_{a,3}$**	**$R_{a,m}$**	**$R_{a,c}$**
[mm]	**[mm]**	**[m/min]**	**[µm]**	**[µm]**	**[µm]**	**[µm]**	**[µm]**
0.1	0.1	200	0.47	0.45	0.48	**0.466**	**0.016**
0.2	0.1	200	0.47	0.48	0.49	**0.480**	**0.063**
0.4	0.1	200	0.54	0.55	0.57	**0.553**	**0.255**
0.6	0.1	200	0.73	0.69	0.74	**0.720**	**0.578**
0.8	0.1	200	1.2	1.39	1.28	**1.290**	**1.034**
1	0.1	200	1.15	1.08	1.14	**1.123**	**1.627**
1.2	0.1	200	2.78	2.86	2.79	**2.810**	**2.361**

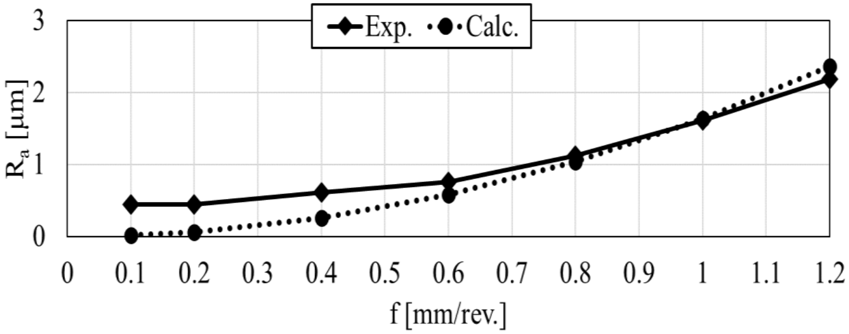

Figure 3 – Measured and calculated values of R_a

Based on the measured and calculated values, if the feed is below 0.6 mm/rev, the difference between the two is proportionally higher than the difference in higher feeds. This caused by that phenomenon that at lower feeds the geometry of the tool has a weaker effect, with the machined surface roughness being influenced more by the material composition, the attributes of the surface layer and the deformation in the cut zone. The latter effects remain nearly constant at different feeds; however, the periodical profile generating effect of the cutting edge geometry becomes dominant. This finding can be clearly seen in Figure 4.

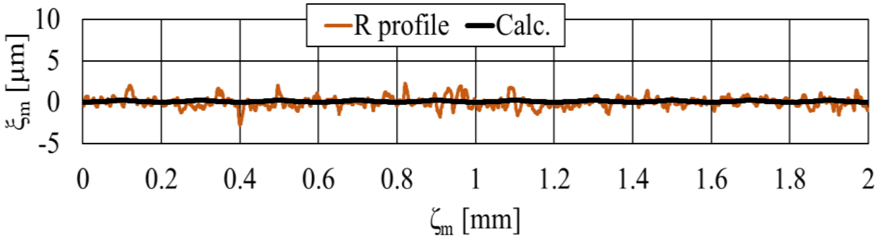

Figure 4a – Measured and calculated roughness profiles (f = 0.2 mm/rev.)

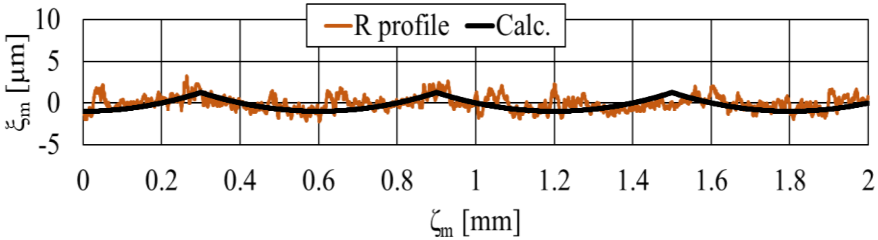

Figure 4b – Measured and calculated roughness profiles (f = 0.6 mm/rev.)

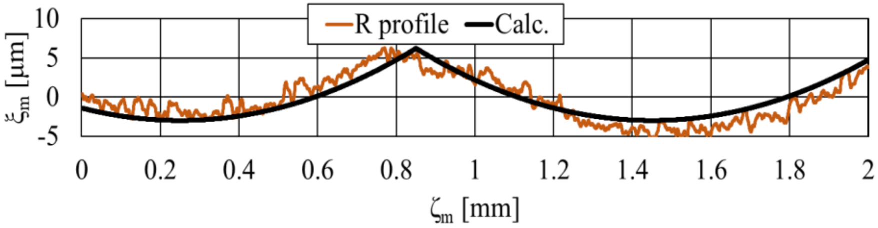

Figure 4c – Measured and calculated roughness profiles (f = 1.2 mm/rev.)

SUMMARY

The effect of feed alteration in rotational turning is studied based on the theoretical and experimental values of Arithmetic Mean Deviation in this paper. We presented a method for the determination of the theoretical value of the analysed roughness value. Based on the equation of the cut surface deducted using by constructive tool geometry, we described the equation necessary for the calculation of R_a in function of the influencing kinematic and geometric parameters. For the chosen technological parameters in the studied range, we calculated the theoretical values, which are validated through cutting experiments.

It was found that the theoretical values of R_a are typically lower than the measured roughness values in the 0.1-0.2 mm/rev feed interval. The change in roughness with the increasing feed is described well by the calculated values, the trends are the same. Roughness profiles are also showed that with the increase of the feed, the periodic profile generating effect of the tool edge geometry becomes dominant, while the proportion of other topography affecting mechanisms becomes lower.

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ТЕОРЕТИЧНА ЗНАЧИМІСТЬ І ЕКСПЕРИМЕНТАЛЬНЕ ДОСЛІДЖЕННЯ СЕРЕДНЄАРИФМЕТИЧНОГО ВІДХИЛЕННЯ ПРИ РОТАЦІЙНОМУ ТОЧІННІ

Анотація. *Наведено метод розрахунку середнього арифметичного відхилення R_a для ротаційного точіння. Наведено необхідні рівняння для розрахунку R_a, виходячи з рівняння теоретичного профілю поверхні різання, та визначено теоретичні значення у досліджуваному діапазоні технологічних параметрів. З цими даними були проведені експерименти з різання та вимірювання значень шорсткості оброблених поверхонь. Потім було проведено порівняльний аналіз виміряних та розрахованих значень середнього арифметичного відхилення. При визначенні шорсткості обробленої поверхні необхідно враховувати, що значення, що теоретично визначаються і практично вимірювані, зазвичай різні, хоча між ними може спостерігатися строга кореляція. Теоретична шорсткість визначається положенням різальної крайки в базовій площині інструменту та подачею, що створює періодичність поверхні. При ротаційному точінні перший визначається радіусом обробленої поверхні (r_m), радіусом (r_s) та кутом нахилу (λ_s) гвинтової різальної крайки. Періодичність поверхні задається обертами інструменту та заготівки або кутовою швидкістю (ω_s і ω_m), додатковою осьовою подачею ($v_{s,a}$), кутом нахилу та радіусом різального інструменту. На основі виміряних і розрахованих значень, якщо подача нижче 0,6 мм/об, різниця між ними пропорційно більша, ніж різниця при більш високих подачах. Це викликано тим явищем, що при менших подачах геометрія інструменту має менший вплив, а на шорсткість поверхні, що обробляється, більший вплив надають склад матеріалу, властивості поверхневого шару і деформації в зоні різання. Останні ефекти залишаються майже постійними при різних подачах; проте переважаючим стає вплив геометрії різальної крайки у створенні періодичного профілю. Було виявлено, що теоретичні значення R_a зазвичай нижчі за виміряні значення шорсткості в інтервалі подач 0,1-0,2 мм/об. Зміна шорсткості зі збільшенням подачі добре описується розрахунковими значеннями, тренди ті ж самі. Профілі шорсткості також показали, що зі збільшенням подачі переважаючим стає ефект формування періодичного профілю геометрії різальної крайки інструменту, тоді як частка інших механізмів, що впливають на топографію, знижується.*

Ключові слова: *середнє арифметичне відхилення; ротаційне точіння; теоретичне та експериментальне дослідження.*