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ANALYTICAL ANALYSIS OF THE THEORETICAL SURFACE ROUGHNESS IN THE CASE OF BURNISHING OF CYLINDRICAL WORKPIECE

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Abstract. *In the last decade, ensuring the highest possible surface quality of manufactured parts has been given a major priority, so more and more emphasis is being placed on the examination and development of finishing treatments that can effectively ensure increasingly stringent surface roughness. For this purpose, diamond burnishing - which is a widely used cold plastic technology - can be used productively as it can improve the surface roughness of the material. But even though due to the development of engineering technology, new possibilities and methods are constantly being developed to examine individual material structure changes, the ability to plan the surface roughness is very difficult. This paper focused on the determination of theoretical roughness to establish a mathematical model that can predict and analyse the relationship between experimental process parameters and surface roughness parameters. To validate the model, real experiments were performed, where the surface roughness were measured before and after the application of burnishing process on low alloyed aluminium shaft pieces.*
Keywords: *mathematical model; modified Hertz theory; contact theory; problems of contact mechanics*.

1. INTRODUCTION

The calculation of theoretical roughness has a long history in the case of cutting operations [1– 6], in longitudinal turning, which is most like the kinematics of surface burnishing as Fig. 1 shows, the theoretical roughness can be determined analytically as a combination of the rotational and axial linear feed of the workpiece.

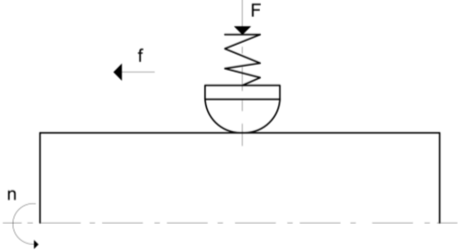

Figure 1. Schematic illustration of burnishing [7]

But the effectiveness of diamond surface burnishing and the plastic deformation process itself are influenced by many parameters, even to a critical degree. For example, inappropriately selected burnishing force can generate lower compressive and, in extreme cases, tensile residual stress, thus reducing the wear resistance and service life of the surface. To optimise the process, many researchers have worked on modelling surface roughness and stress state, which can be used to estimate the burnishing efficiency of a given material grade.

Cui et al. [8] found that their model based on Hertz theory can be used as a good approach for estimating surface roughness when applying excessive burnishing forces to Inconel 718 material grade. Vaishya et al. [9] also started from the use of the Hertz theory in the case of machining Stavax material quality with TiC burnishing tool and primarily investigated the effect of the burnishing force. The results of the experimental and theoretical values showed a good approximation and according to their claim, the deviation was caused by just the assumptions made during the development of the model, however, these uncertain factors were not named. Felhő and Varga [10] determined the theoretical roughness for diamond burnishing of 30CrMo4 material, which they compared with both finite element modelling and real experimental results, and their work was greatly influenced by the modified Hertz theory of Bouzid et al. [11].

In his work, Korzynksi [12] compared several models applicable to surface burnishing and showed that classical burnishing models are only valid for so called surface hardening burnishing. They are suitable for determining the tool indentation depth, but do not consider the stereometric condition of the initial surface, which has a significant influence on the surface roughness after burnishing. Felhő [13] presents in his publication the possibilities of analytical modelling of theoretical roughness for different machining processes. He points out also one of the obvious disadvantages of these calculations is that they ignore many factors that can influence the real surface roughness: the cutting/burnishing forces and the associated vibrations of the machining system, the roughness of the tool, etc.

To explain the contact mechanical relations between bodies with circular surfaces (sphere, cylinder), several models have been developed, such as Johnson-Kendall-Roberts model, Bradley model, Derjaguin-Müller-Toporov model, but based on my experience in literature research, the most preferred solution is to use Hertz theory, so the relation developed for normal contact of elastic solids. In this paper, the approach of Ponomarjov et al. [14] with Hertz-theory is combined to determine the theoretical roughness of burnished EN AW-2011 cylindrical workpiece.

2. ANALYTICAL MODEL OF SURFACE ROUGHNESS

The contact of deformable solids is a common phenomenon in nature and in engineering practice, and plays an important role in physics, biology, astrophysics, nanoindentation, etc. [15, 16]. But the examination of deformations and stresses at the point of contact of the components is one of the most complicated chapters in modern elasticity. The theory of deformation of elastic bodies was founded by Hertz and it has been a milestone in modern contact mechanics since it was published publicly in 1882. It describes the contact mechanism of two solid bodies in the range of linear elasticity and negligible deformations [14].

In the case of burnishing process, the use of this theory is justified because burnishing is a chipless manufacturing technology, so the depth of cut as a parameter cannot be interpreted. However, in order to analytically determine the theoretical roughness, it is necessary to calculate the so-called normal indentation depth of the tool (Figure 2 – δ), as this already enables the use of the modelling method which is correctly applied in turning. Figure 2 schematically shows that during the burnishing process, the tool with the radius "R_1" penetrates the material of the workpiece and generates an indentation depth "δ" in the axial width "2a" under the influence of the force "*F*".

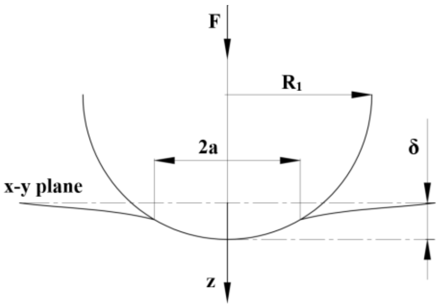

Figure 2. Theoretical illustration of surface burnishing

Hereinafter, I briefly summarize the treatment of the spherical burnishing tool and the outer axial workpiece with Hertz-theory, which is also based on the thought process of Johnson-Kendall-Roberts and Ponomarjov.

According to the Hertz contact theory the radii of the two contacting balls are R_1 and R_2 respectively, it can be seen on the left side in Figure 3. On the right side of it, "r" denotes the distance of the moving point from the force, "w_1" and "w_2" denote the displacement of the bodies in the directions "z_1" and "z_2", then the convergence of the given points can be described as follows:

$$\delta = (z_1 + w_1) + (z_2 + w_2) \tag{1}$$

In this way, contact the points for which it is satisfied that:

$$z_1 + z_2 = \delta - (w_1 + w_2) \tag{2}$$

From Fig. 3:

$$R_1^2 - r^2 = (R_1 - z_1)^2 = R_1^2 - 2R_1 r + {z_1}^2 \tag{3}$$

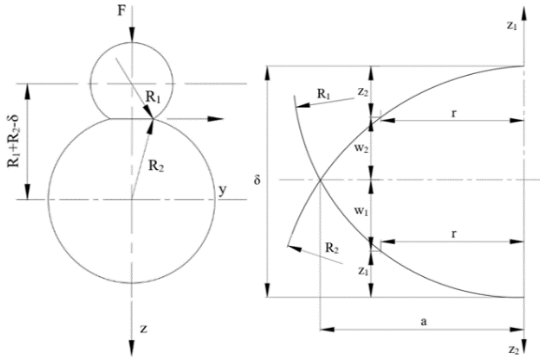

Figure 3. Approach distance of contacting bodies

"${z_1}^2$" is a second-order small value, negligible, thus:

$$z_1 = \frac{r^2}{2R_1}\,;\, z_2 = \frac{r^2}{2R_2} \tag{4}$$

The distance of the approaching points after contact can be expressed as follows:

$$w_1 + w_2 = \delta - (z_1 + z_2) = \delta - \frac{r^2}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{5}$$

As Fig. 4 shows, when machining an axial workpiece with a spherical tool, the surface pressure is distributed over an ellipsoidal surface and is proportional to the ordinates of the ellipsoid.

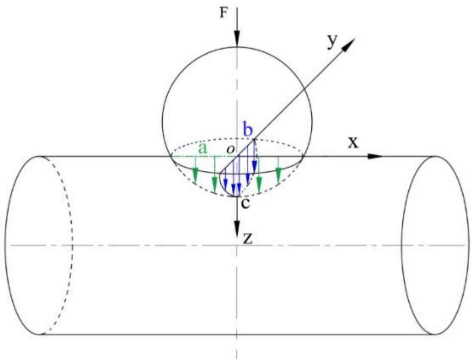

Figure 4. Pressure distribution during burnishing axial workpiece

The equation of ellipsoid must be applied:

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2 = 1 \tag{6}$$

As the pressure distribution on the contact area is axisymmetric about x-y axes and the maximum pressure is located at the centre of the contact area (O), the pressure distribution supposed to be semi-ellipsoid, so:

$$w_1 + w_2 = \delta - \frac{x^2}{2R_1} - \frac{y^2}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{7}$$

Under this semi-ellipsoidal load, on S_c contact surface the vertical displacement of the elliptical contact point “w” can be expressed as follows using the elastic material parameters:

$$w_1 = k_1 \int_{S_c} \frac{p}{r} dS \; ; w_2 = k_2 \int_{S_c} \frac{p}{r} dS \tag{8}$$

Relation for the calculation of elastic material parameters "k_1" and "k_2":

$$k_1 = \frac{1-\nu_1^2}{E_1} \; ; k_2 = \frac{1-\nu_2^2}{E_2} \tag{9}$$

While, according to Fig. 4, the pressure distribution can be obtained as (10):

$$p = p_0 \frac{c}{a} = p_0 \sqrt{1 - \left(\frac{x}{a}\right)^2 - \left(\frac{y}{a}\right)^2} \tag{10}$$

And, given the corresponding force balance equation:

$$p_0 = \frac{3F}{2ab\pi} \tag{11}$$

Based on the relationships described so far, the calculation for major (a) and minor semi-axis (b) of the ellipse and the value of the approach distance [14–17]:

$$a = \alpha \sqrt[3]{\frac{3}{4} \frac{2R_1R_2}{R_1+2R_2} \left(\frac{1-\vartheta_1^2}{E_1} + \frac{1-\vartheta_2^2}{E_2}\right)^2 F} \tag{12}$$

$$b = \beta \sqrt[3]{\frac{3}{4} \frac{2R_1R_2}{R_1+2R_2} \left(\frac{1-\vartheta_1^2}{E_1} + \frac{1-\vartheta_2^2}{E_2}\right)^2 F} \tag{13}$$

$$\delta = \gamma \sqrt[3]{\frac{9}{128} \frac{R_1+2R_2}{2R_1R_2} \left(\frac{1-\vartheta_1^2}{E_1} + \frac{1-\vartheta_2^2}{E_2}\right)^2 F^2} \tag{14}$$

α, β and γ are coefficients, which can be calculated from Table 1 [16] with equation (15):

$$cos\theta \ p = \frac{R_1}{R_1 + 2R_2} \quad (15)$$

Table 1. Table of coefficients values [18]

θ	0°	10°	20°	30°	35°	40°	45°	50°
α	∞	6.612	3.778	2.397	2.397	2.136	1.926	1.754
β	0	0.319	0.408	0.493	0.530	0.576	0.604	0.641
γ	-	0.851	1.220	1.550	1.550	1.637	1.709	1.772
θ	**55°**	**60°**	**65°**	**70°**	**75°**	**80°**	**85°**	**90°**
α	1.611	1.486	1.378	1.284	1.202	1.128	1.061	1.00
β	0.678	0.717	0.759	0.802	0.846	0.893	0.944	1.00
γ	1.828	1.875	1.912	1.944	1.967	1.985	1.996	2.00

Table 2 summarizes the applied parameters and the calculated values for this idealised elliptical contact area.

Table 2. Applied parameters and results of analytical calculation

F **[N]**	***R₁*** **[mm]**	***R₂*** **[mm]**	**ν₁**	**ν₂**	***E₁*** **[N/mm²]**	***E₂*** **[N/mm²]**	**Θ (°)**	**δ [μm]**
20	3.5	21.5	0.07	0.33	$11.43 \cdot 10^5$	$7 \cdot 10^4$	85	
α (°)	**β (°)**	**γ (°)**	***k₁***		***k₂***	***a*** **[mm]**	***b*** **[mm]**	**2.34**
1.061	0.944	1.996	$2.77 \cdot 10^{-7}$		$4.04 \cdot 10^{-6}$	0.0924	0.0822	

The value of the roughness height factor (R_t) resulting from the application of surface burnishing is influenced by the surface roughness created by the previous operation, the burnishing feed (v_f), the penetration depth of the burnishing tool (δ), and the so-called furrow depth (h), which represents the maximum of the theoretical roughness peaks created during burnishing, as illustrated in Figure 5 [11, 18].

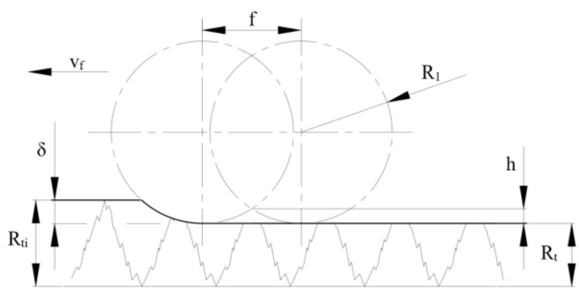

Figure 5. The relationship between the theoretical maximum furrow depth and the indentation depth when using a small burnishing feed [11]

This factor is determined by calculating the intersection point of two tool impressions that are one unit feed apart – assuming that its value is smaller than the normal indentation depth - according to formula (16).

$$h = \frac{125f^2}{R_1} \tag{16}$$

Knowing the normal indentation depth and the furrow depth, the theoretical maximum roughness can be determined, so the distance between the highest and lowest point of the profile:

$$R_t = R_{t_i} - \delta + h \tag{17}$$

3. REAL MACHINING EXPERIMENTS

For validating theoretical investigation, the real burnishing experiment were carried out on EN AW-2011 low alloyed aluminium shaft. Burnishing operations were preceded by finishing turning set at f_1 = 0.2 mm/rev and then f_2 = 0.15 mm/rev feed rate with an E400 universal lathe. Then, the burnishing was carried out with the same machine using a PCD spherical burnishing tool (r = 3.5 mm) and the kinematic viscosity of the applied manual dosing oil was ν = 70 mm^2/s, while the burnishing speed was v = 50.54 m/min, the burnishing force F = 20 N and feed rate is v_f = 0.001 mm/rev.

Surface roughness measurements – before and after burnishing – were conducted on an Altisurf 520 3D surface topography measuring device. A CL2 confocal chromatic sensor was used, the cut-off was 0.8 mm and Gauss filter was applied. The measurement results were evaluated with Altimap Premium.

4. RESULTS AND DISCUSSION

This publication dealt with the analytical determination of the surface roughness generated by diamond burnishing applied to the external cylindrical surface. Considering the many uncertainty factors (stiffness of the machine system, vibrations, roughness of the tools, etc.), the author tried to apply the correct mathematical model in the case of the contact mechanics problem of spherical and cylindrical bodies, studying the Hertz- and Jhonson-Kendall-Roberts-theory, combining it with Ponomarjov's approach. The value obtained during the theoretical determination of the indentation depth of the tool– which is of prime importance –

was 2.34 μm, while in the real case it was 3.62 μm. The two values come close enough to form the basis of further theoretical and experimental investigations.

It is important to note that the procedure in question and the related analytical models were applied primarily in the case of metallic material qualities, while the present study focuses on the examination of non-metallic material.

My future plans include using the analytical model to create a finite element simulation model and then validating these results with real burnishing experiments.

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АНАЛІТИЧНИЙ АНАЛІЗ ТЕОРЕТИЧНОЇ ШОРСТКОСТІ ПОВЕРХНІ В РАЗІ ВИГЛАДЖУВАННЯ ЦИЛІНДРИЧНОЇ ЗАГОТОВКИ

Анотація. *В останнє десятиліття забезпечення максимально можливої якості поверхні виготовлених деталей стало основним пріоритетом, тому все більше уваги приділяється вивченню та розробці фінішної обробки, яка може ефективно забезпечити найнижчу шорсткість поверхні. Для цієї мети можна продуктивно використовувати алмазне вигладжування, яке є широко використовуваною технологією холодної пластичної деформації, оскільки воно може покращити шорсткість поверхні матеріалу. Але, незважаючи на те, що у зв'язку з розвитком інженерних технологій постійно розробляються нові можливості і методи вивчення індивідуальних змін структури матеріалу, можливість планування шорсткості поверхні дуже ускладнена. Ця робота була присвячена визначенню теоретичної шорсткості для створення математичної моделі, яка може передбачати та аналізувати взаємозв'язок між параметрами експериментального процесу та параметрами шорсткості поверхні. Для валідації моделі були проведені реальні експерименти, під час яких вимірювалася шорсткість поверхні до і після застосування процесу вигладжування на низьколегованих алюмінієвих валах. Враховуючи безліч факторів невизначеності (жорсткість системи верстата, вібрації, шорсткість інструментів тощо), авторка спробувала застосувати правильну математичну модель у випадку задачі контактної механіки сферичних і циліндричних тіл, вивчаючи теорію Герца і Джонсона-Кендалла-Робертса, поєднуючи її з підходом Пономарьова. Значення, отримане при теоретичному визначенні глибини вдавлення інструменту, що має першорядне значення, становило 2,34 мкм, тоді як у реальному випадку – 3,62 мкм. Ці дві величини досить близькі, щоб лягти в основу подальших теоретичних і експериментальних досліджень. Важливо відзначити, що дана процедура і пов'язані з нею аналітичні моделі були застосовані в основному у випадку якості металевих матеріалів, в той час як дане дослідження зосереджено на дослідженні неметалічних матеріалів. Подальші плани авторки включають використання аналітичної моделі для створення моделі імітації скінченних елементів, а потім перевірку цих результатів за допомогою реальних експериментів з поліруванням.*

Ключові слова: *математична модель; модифікована теорія Герца; теорія контакту; задачі контактної механіки.*