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IMPACT OF QUANTITY DISCOUNT ON PURCHASING COSTS IN BLENDING TECHNOLOGIES

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Abstract. *Blending technologies are important in many sectors of industry, but are most prevalent in the chemical and food industries. They are playing an increasingly important role in the world economy despite the spread of electromobility. Nowadays, in addition to the technological aspects, there is a growing need to look at the logistical aspects, as logistics related costs account for a significant part of the cost of blending technologies. In this research work, the results of the analysis of the impact of quantity discounts, an important aspect of procurement activities related to blending technologies, are presented. A mathematical model is presented that can be used to investigate the impact of quantity discounts of components on profit and product quality. Based on scenario analyses carried out based on the mathematical model, it is demonstrated that the quantity discount can have a significant impact not only on total cost and profit, but also on the quality of the finished blended product.*

Keywords: *blending technologies; cost efficiency; optimisation; quantity discount; product quality.*

1. INTRODUCTION

The chemical and food industries typically use blending technology to produce their products. These blending technologies are becoming increasingly widespread, producing large quantities of high value products. The global fossil fuel consumption in 2022 was 39.413 TWh gas, 52.970 TWh oil and 44.854 coal [1]. This huge amount of fossil energy is generally processed by blending technologies, therefore it is important to improve both technological and logistics processes and blending technologies. Research of the technology and logistics associated with blending technologies is very significant, but the integrated study of technological and logistical aspects is still a less researched area.

The logistics aspects of blending technologies affect procurement, production, distribution and recycling, while manufacturers and service providers have to face both the positive and negative impacts of the fourth industrial revolution, globalisation and digitalisation. In this research paper, the author aims to investigate a fundamental area of procurement logistics. The aim of the research is to develop a mathematical model to investigate the effects of quantity discounts, well known in procurement logistics, on the quality of products produced using blending technologies. The research work discusses an important topic of blending technologies, because logistics represents an important part of the total costs.

This paper is organized as follows. Section 2 shows the most important research results from both technological and logistics point of view. Section 3 proposes a mathematical model, which makes it possible to analyse the impact of quantity discount of components on the total cost, profit and product quality. Section 4 discusses the results of the numerical analysis of two scenarios, which validates the mathematical model. Conclusions, future research directions and managerial impacts are discussed in Section 5.

2. LITERATURE REVIEW

The research results regarding blending technologies can be divided into two main parts. The first part focuses on technological aspects of blending technologies, while the second part discusses logistics aspects in blending technologies.

The technological researches include a wide range of quality and processing related aspects, for example development of diesel fuel blending technologies using house of quality [2], fusibility and gasification aspect in coal blending technologies [3], automation and digitalization of cotton blending technologies [4], development of novel blending technologies for power generation focusing on biomass blending [5]. As the technological researches shows, the development of blending technologies is important not only from financial aspects, but it has a significant impact on environment [6]. The blending technology can also be researched from stability of final product point of view and stability can be significantly influenced by components [7]. Blending technologies and blending processes are complex, their optimisation can lead to complex NP-hard optimisation problems and the solution of these NP-hard optimisation problems is generally possible by heuristics and metaheuristics, as Cai et al. shows in a research focusing on optimisation of coal blending technologies using quantum particle swarm optimization [8]. Other interesting research works are focusing on online blending [9], mineral processing [10], intensification of flow blending technologies [11], blending technologies and renewable fuel [12], stability in blending [13], extraction processes [14] and melt blending technologies [15].

The logistics aspects of blending technologies include purchasing logistics, production logistics, distribution logistics and recycling logistics, but the mainly researched area is purchasing logistics, because a suitable optimised supply chain can significantly increase the efficiency of blending technologies. The supplier selection problem is extensively discussed in [16]. The author focuses on the global supply chain and discusses the potentials in cross-docking facilities to improve availability and efficiency of material supply in blending technologies. A integrated approach is discussed in [17] focusing on stock size, batch size, transportation, storage and supply chain structure. The networking potentials of blending technologies are also discussed by researchers [18]. Outsourcing is also an important potential in logistics processes of blending technologies, as a research validates the importance of outsourcing activities in supply chain solutions [19]. However, the mathematical modelling generally leads to complex, NP-hard models, but there are approaches, which shows, that LP models can be also suitable to model logistics aspects of blending technologies [20].

As this short literature review shows, the research of blending technologies is important both from technological and logistics aspects. Within the frame of this article, a purchasing logistics related approach is discussed to show the importance of quantity discount in purchasing and procurement decisions.

3. MATERIALS AND METHODS

Within the frame of this part of the article, the mathematical model for the analysis of the impact of quantity discounts on the purchasing costs for blending technologies is discussed.: The input parameters of the optimization model are the followings:

- specific price of blended products: p_j^*, where $j = 1 \dots j_{max}$,
- specific purchasing price of component *i* for the final products: $p_i(a_i^*)$, where

$$a_i^* = \sum_{i=1}^{i_{max}} a_{ij} \quad (1)$$

and a_{ij} defines the amount of component *i* assigned to final product *j*,
- specific blending cost of component *i* into final product *j*: c_{ij}^T
- quality parameter *k* of component *i*: μ_{ik},
- demand for final product *j*: d_j,
- lower limit for the quality parameter *k* for final product *j*: μ_{jk}^{MIN},
- upper limit for the quality parameter *k* for final product *j*: μ_{jk}^{MAX},
- the total available inventory of component *i* available to be purchased: a_i^{max}.

The objective function of the optimisation of the blending processes is generally a cost function, which defines the minimization of the total cost depending on the specific price of blended products, the purchasing price of components, the amount of purchased components required to blend the final products, and the specific blending cost as follows:

$$C = \sum_{j=1}^{j_{max}} p_j^* + \sum_{i=1}^{i_{max}} \sum_{j=1}^{j_{max}} a_{ij} \cdot \left(p_i(a_i^*) + c_{ij}^T\right) \rightarrow min. \quad (2)$$

The first constraint defines, that it is not allowed to exceed the available upper order limit, which depends on the available component inventory of suppliers:

$$\forall i: \sum_{j=1}^{j_{max}} a_{ij} \leq a_i^{max} \quad (3)$$

The second constraint defines, that it is not allowed to exceed the lower and upper limit of quality parameters predefined for the blended final products:

$$\forall j,k: \mu_{jk}^{MIN} \leq \sum_{i=1}^{i_{max}} a_{ij} \cdot \mu_{ik} \cdot \left(\sum_{i=1}^{i_{max}} a_{ij}\right)^{-1} \leq \mu_{jk}^{MAX} \tag{4}$$

The third constraint defines, that it is not allowed to exceed the available component specific technological resources:

$$\forall i: \sum_{j=1}^{j_{max}} f_{ij}^{T} \leq f_i^{TMAX} \tag{5}$$

Within the frame of this model, the impact of specific purchasing price is discussed, therefore it is important to define the specific price depending on the order quantity. The first way to define the purchasing price depending on the order quantity is a price function, as follows:

$$\forall i: p_i(a_i^*) = p_i^B - \alpha^{a_i^*} \tag{6}$$

where p_i^B is the initial purchasing cost of component i, and α is a discount factor. The second way to define the purchasing price depending on the order quantity is a stepwise function as follows:

$$\forall i: p_i(a_i^*) = \begin{bmatrix} a_i^* \leq h^{a1} \rightarrow & p_i(a_i^*) = p_i^{a1} \\ h^{a1}+1 < a_i^* \leq h^{a2} \rightarrow & p_i(a_i^*) = p_i^{a2} \\ \dots & \dots \\ h^{g-1}+1 < a_i^* \leq h^{ag} \rightarrow & p_i(a_i^*) = p_i^{ag} \end{bmatrix} \tag{7}$$

The third way define to define the purchasing price depending on the order quantity is a capacity related function, which defines the specific purchasing price increased by the logistics related costs. One typical type of these logistics related costs is the specific transportation cost depending on the capacity of the transportation resources. Figure 1 shows examples for these three types of specific purchasing price functions.

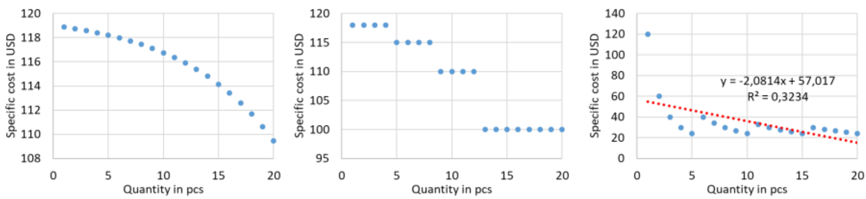

Figure 1 – Specific cost models

The decision variable of the optimization problem is the $A = [a_{ij}]$ assignment matrix, which defines that a_{ij} quantity from component i has to be purchased for final product j. The supplier selection problem is also an important part of purchasing strategies regarding blending technologies, within the frame of this article the single supplier solution is discussed, but the models are also suitable for multi supplier solution, because it is possible to define products from different suppliers as different products.

4. RESULTS

Within the frame of this chapter, two scenarios will be described to validate the proposed optimization approach to find the optimal purchasing quantity of components for the final products to be blended. Scenario 1 discusses the impact of quantity discount of components on the total price and profit, while scenario 2 focuses on the impact of quantity discount on the quality parameters of final products. In this scenarios, the capacity constraints are not taken into consideration, but in a future research, the connection between constrained available technological resources and quantity discount of components can be also analysed. The scenarios include 5 final products and 5 components, each having 3 quality parameters, which are defined as proportions in percent. All quality parameters have lower and upper limits, but in practical cases, the quality parameters can be constrained either by a lower or by an upper limit and it is not allowed to exceed this lower or upper limit.

Scenario 1: Numerical analysis of a case study focusing on costs

Within the frame of this scenario, the impact of quantity discount on the total cost and profit are described, while quality and capacity related constraints are taken into consideration. Table 1 summarizes the input parameters of components.

Table 1 – Input parameters of components in scenario 1

Input parameters of components	Notation	Component ID				
		C1	C2	C3	C4	C5
Initial purchasing price in USD	p_i^B	3	4	5	6	7
Quantity specific purchasing price	$p_i(a_i^*)$	$p_i^B - (\sum_{j=1}^{j_{max}} d_j)^{-1} \sum_{j=1}^{j_{max}} a_{ij}$				
Quality parameter 1 in %	μ_{i1}	10	12	14	11	9
Quality parameter 2 in %	μ_{i2}	1	2	3	2	4
Quality parameter 3 in %	μ_{i3}	30	34	41	29	52
Available amount in pcs	a_i^{max}	150	20	260	50	150

Table 2 shows the input parameters of final products to be blended.

Table 2 – Input parameters of final products in scenario 1

Input parameters of final products	Notation	Product ID				
		A1	A2	A3	A4	A5
Demand in pcs	d_j	100	90	80	110	55
Specific price in USD	p_j^*	10	10	10	10	10
Lower limit of quantity parameter 1 in %	μ_{j1}^{MIN}	9	9	9	9	9
Upper limit of quantity parameter 1 in %	μ_{j1}^{MAX}	14	14	14	12	14
Lower limit of quantity parameter 2 in %	μ_{j2}^{MIN}	2	2	3	2	2
Upper limit of quantity parameter 2 in %	μ_{j2}^{MAX}	3	4	4	4	3

Lower limit of quantity parameter 3 in %	μ_{j3}^{MIN}	30	34	30	40	30
Upper limit of quantity parameter 3 in %	μ_{j3}^{MAX}	35	52	45	52	40

Based on Table 1 and Table 2, the specific purchasing cost for each component can be described as follows:

- for components C1: $p_1(a_1^*) = 3 - 435^{-1}\sum_{j=1}^{5} a_{1j}$,
- for components C2: $p_2(a_2^*) = 4 - 435^{-1}\sum_{j=1}^{5} a_{2j}$,
- for components C3: $p_3(a_3^*) = 5 - 435^{-1}\sum_{j=1}^{5} a_{3j}$,
- for components C4: $p_4(a_4^*) = 6 - 435^{-1}\sum_{j=1}^{5} a_{4j}$,
- for components C5: $p_5(a_5^*) = 7 - 435^{-1}\sum_{j=1}^{5} a_{5j}$.

The above described scenario is a non-linear optimization problem, which can be solved by nonlinear regression. The optimization led to the results shown in Table 3.

Table 3 – Resulted assignment matrix of the scenario describing the amount of components to be purchased in pcs in scenario 1

Component ID	Product ID				
	A1	A2	A3	A4	A5
C1	46	45	0	30	27
C2	0	0	0	0	0
C3	46	45	80	60	28
C4	8	0	0	0	0
C5	0	0	0	20	0

The quality parameters of final products are shown in Figure 2. As Figure 2 shows, all quality parameters are between the predefined lower and upper limit.

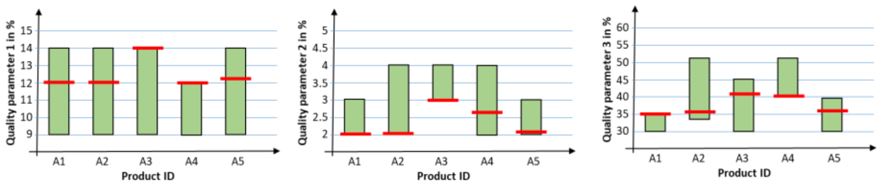

Figure 2 – Quality parameters of final products in scenario 1

In this scenario, the total cost is 1748.1 USD, while the maximized profit is 2601.9 USD. In the case of model without quantity discount, the total purchasing cost would be 178.9 USD higher.

Scenario 2: Comparison of quality parameters with and without quantity discount

In the next scenario, the impact of the quantity discount on the optimized product quality parameters are discussed. As shown in the first scenario, the quantity discount of components has a significant impact on total cost and profit, but it is also important to analyse the impact of quantity discount on quality parameters of final products, as quantity discount may make it worthwhile to order more of certain components, which may lead to a decrease in the quality parameters of the finished products.

Table 4 shows the lower and upper limit of quality parameters for final products. The demands and the specific prices of final products are the same as in the case of scenario 1.

Table 4 – Lower and upper limit of quality parameters for final products in scenario 2

Input parameters of final products	Notation	Product ID				
		A1	A2	A3	A4	A5
Lower limit of quantity parameter 1 in %	μ_{j1}^{MIN}	10	9	7	9	6
Upper limit of quantity parameter 1 in %	μ_{j1}^{MAX}	20	18	20	20	17
Lower limit of quantity parameter 2 in %	μ_{j2}^{MIN}	2	2	3	2	2
Upper limit of quantity parameter 2 in %	μ_{j2}^{MAX}	5	5	5	5	5
Lower limit of quantity parameter 3 in %	μ_{j3}^{MIN}	30	31	34	40	35
Upper limit of quantity parameter 3 in %	μ_{j3}^{MAX}	55	60	50	45	55

Table 5 summarizes the input parameters of components.

Table 5 – Input parameters of components in scenario 2

Input parameters of components	Notation	Component ID				
		C1	C2	C3	C4	C5
Initial purchasing price in USD	p_i^B	10	12	20	8	11
Quality parameter 1 in %	μ_{i1}	10	12	14	11	9
Quality parameter 2 in %	μ_{i2}	1	2	3	2	4
Quality parameter 3 in %	μ_{i3}	30	34	41	29	52
Available amount in pcs	a_i^{max}	200	120	50	350	200

The optimization of scenario 2 led to the results shown in Table 6.

Table 6 – Resulted assignment matrix of the scenario describing the amount of components to be purchased in pcs in scenario 2

Component ID	Product ID				
	A1	A2	A3	A4	A5
C1	0	0	0	0	0
C2	0	0	0	0	0
C3	0	0	0	0	0
C4	95	82	40	57	40
C5	5	8	40	53	15

The quality parameters of final products are shown in Figure 3. As Figure 3 shows, all quality parameters are also between the predefined lower and upper limit.

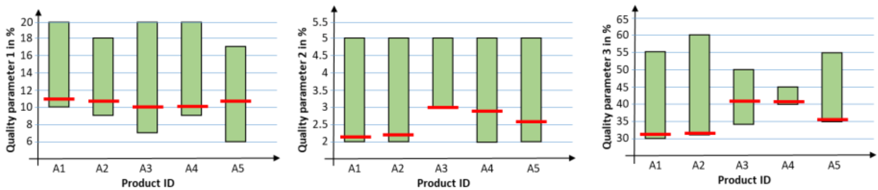

Figure 3 – Quality parameters of final products in scenario 2

In scenario 2, the total cost is 3843 USD, while the maximized profit is 507 USD. Using the following quantity discount model for scenario 2:

$$\forall i: p_i(a_i^*) = p_i^B - \alpha^{a_i^*} \quad (8)$$

where $p_i^B = (10,12,20,8,11)$ and $\alpha = (1.01, 1.001, 1.01, 1.01, 1.005)$, the total profit is 2009.4 USD and the total cost is 2340.6 USD.

In this second part of scenario 2, the quality parameters are also between the lower and upper limits, as shown in Figure 4.

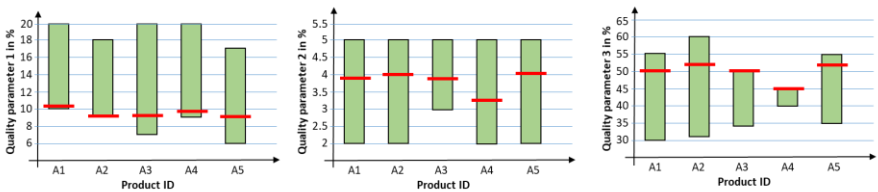

Figure 4 – Quality parameters of final products in scenario 2

As the comparison of quality parameters with and without quantity discount shows, the quantity discount has significant impact on the quality of blended final products. Figure 5 summarizes the differences of quality parameters with and without quantity discount.

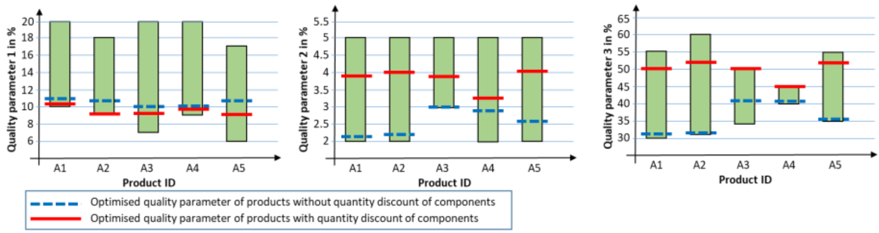

Figure 5 – Comparison of quality parameters of final products in scenario 2 with and without quantity discount of components

As Figure 6 shows, the proportion of optimised quality parameters with and without quantity discount of components can be different, which means, that in the case of scenario 2, quality parameter 1 was lower, while quality parameters 2 and 3 were higher with quantity discount.

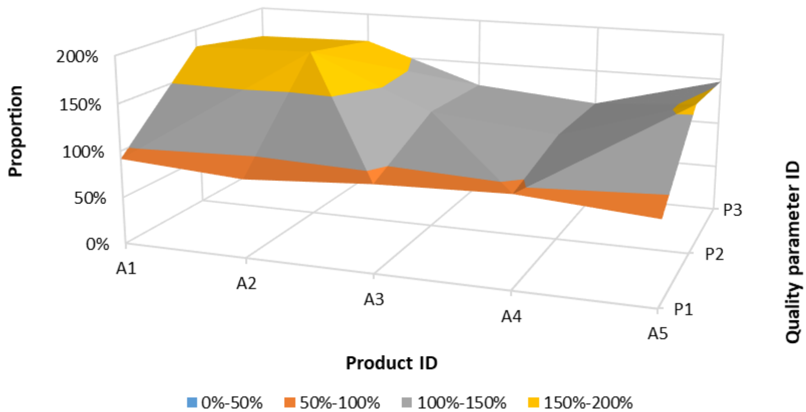

Figure 6 – Proportion of optimised quality parameters in scenario 2 with and without quality discount

5. SUMMARY

Blending technologies are a special field of industrial technologies where the quality parameters of the components significantly determine the quality of the final product, and this significant relationship can usually be defined by means of specific mathematical functions. As evidenced by a number of literature sources and research works, one of the most important research directions for blending technologies is the investigation of the technological aspects of blending technologies, while there are only few sources in the literature that deal with the logistical aspects of blending technologies. In the present research work, a mathematical model was developed to investigate how the quantity discount on the components that make up the finished product affects the total cost, profit and the impact on the quality and quality parameters of the finished product. Based on the two case studies presented, it can be concluded that the quantity discount has a significant impact on profit, has an impact on the quantity and quality of the components purchased and thus affects the quality of the finished product.

As a future research direction, it is possible to analysed the impact of dynamically changing specific component costs and quantity discounts on the total cost of blending technologies and various purchasing strategies can be defined depending on the complexity of the products, available components, suppliers and cost models.

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ВПЛИВ КІЛЬКІСТНОЇ ЗНИЖКИ НА ЗАКУПІВЕЛЬНІ ВИТРАТИ В ТЕХНОЛОГІЯХ ЗМІШУВАННЯ

Анотація. *Технології змішування (купажування) важливі в багатьох галузях промисловості, але найбільш поширені в хімічній та харчовій промисловості. Вони відіграють все більш важливу роль у світовій економіці, незважаючи на поширення електромобільності. Сьогодні, окрім технологічних аспектів, зростає потреба розглянути логістичні аспекти, оскільки витрати, пов'язані з логістикою, становлять значну частину собівартості технологій змішування. Купажні технології є особливою галуззю промислових технологій, де параметри якості компонентів суттєво визначають якість кінцевого продукту, і цей суттєвий взаємозв'язок, як правило, може бути визначений за допомогою певних математичних функцій. Як свідчить низка літературних джерел та науково-дослідних робіт, одним з найважливіших напрямків досліджень технологій змішування є дослідження технологічних аспектів технологій змішування, в той час як логістичним аспектам технологій змішування в літературі присвячені лише поодинокі джерела. У даній науково-дослідній роботі була розроблена математична модель для дослідження того, як кількісна знижка на компоненти, що входять до складу готового продукту, впливає на загальну собівартість, прибуток і вплив на параметри якості готового продукту. На основі двох представлених тематичних досліджень можна зробити висновок, що кількістна знижка має значний вплив на прибуток, впливає на кількість та якість придбаних компонентів і, таким чином, впливає на якість готової продукції. В якості майбутнього напрямку дослідження може бути аналіз впливу динамічної зміни питомих витрат на компоненти та кількістних знижок на загальну вартість технологій змішування, а також визначити різні стратегії закупівель залежно від складності продукції, доступних компонентів, постачальників та моделей витрат.*

Ключові слова: *технології змішування; економічна ефективність; оптимізація; кількістна знижка; якість продукції.*