UDC 531.67, 621.923 **doi: 10.20998/2078-7405.2025.102.02**

# MODELING THE IMPACT OF NONLINEAR OSCILLATIONS ON THE QUALITY OF THE WORKING SURFACE OF PARTS IN FINISHING OPERATIONS

Anatoliy **Usov** [0000-0002-3965-7611], Maksym **Kunitsyn** [0000-0003-1764-8922]

National University "Odessa Polytechnic", Odessa, Ukraine
usov_a_v@op.edu.ua

**Received: 22 April 2025 / Revised: 29 April 2025 / Accepted: 20 May 2025 / Published: 20 June 2025**

***Abstract.*** *The paper investigates the influence of finishing operations on the roughness of machined surfaces in the case when the machine-tool-fixture-tool-part (MTFTP) system is in the zone of nonlinear oscillations. For this purpose, models of dynamic oscillatory processes accompanying the machining of working surfaces of parts are built in the Simulink system of the MATLAB package. The formation of self-excited oscillations of the MTFTP mechanical system during finishing operations is determined with one or two degrees of freedom associated with the heterogeneity of the processed material and external disturbing forces in the ranges of fundamental and subharmonic resonances containing, in addition to the excitatory element itself, also zones of dry friction of the tool with the processed surface. These studies not only demonstrate the behavior of mechanical systems capable of self-excited oscillations, but also allow their results to be successfully applied to optimize the quality characteristics of machined surfaces during finishing operations. It has been shown that resonant curves under nonlinear oscillations of mechanical systems of finishing operations affect not only the formation of the roughness of the machined surface, but also the appearance of scorch marks on them and the formation of defects such as cracks.*
***Keywords:*** *finishing operations; nonlinear oscillations; models; Simulink system; oscillation amplitudes; stability.*

## 1. Introduction

Unlike an ideal surface, the surface of a part after finishing operations is not smooth, but always has microscopic irregularities that form roughness [1], [2]. Despite the rather small size of the roughness-forming irregularities, they have a significant impact on various operational properties of parts [3], [4], [5]. In the operations preceding grinding, the roughness of the machined surface affects the concentration of stresses, vibration activity, and the formation of thermal defects, which, under the influence of thermomechanical phenomena accompanying finishing operations, form cauterization, cracks, and chips on the machined surfaces [6], [7]. Numerous theoretical and experimental studies have been devoted to the study of this process [8], [9], [10]: the effect of grinding wheel wear, the creation of macro- and microrelief of the ground surface, fluctuations in thermomechanical parameters in the contact zone of the grinding wheel with the treated surface, and vibration of the machine-tool, fixture, tool, and part (MTFTP) system.

## 2. Analysis of sources and problem description

Let us consider the influence of finishing operations on the roughness of machined surfaces in the case where the MTFTP system is in the zone of nonlinear oscillations. To do this, consider modeling dynamic oscillatory processes in the Simulink system of the MATLAB package [11].

The high speed of abrasive wheels when processing the working surfaces of machine parts highlights the need for the most accurate calculation of their dynamic properties, i.e., the calculation of their natural frequencies, resonance curves, maximum amplitudes, transfer functions, areas of instability, and amplitudes of self-excited vibrations in unstable areas.

The presence of nonlinear couplings, such as, for example, the gap in the connection of individual elements of the MTFTP, contact deformations, nonlinear damping forces, stochastic nature of external influences, etc., leads to the fact that existing classical methods of linear dynamics do not take into account nonlinear oscillations, and there is a need to study the dynamic properties of mechanical systems using nonlinear mechanics [12].

The formation of the quality characteristics of machined surfaces when the mechanical system of the MTFTP falls into the field of nonlinear oscillations is currently relevant.

Theoretical methods for solving these problems are approximate, they are based mostly on certain assumptions, and therefore it is almost always necessary to check the adequacy of theoretical calculations and the limits of applicability of the results obtained on models, both mathematical and simulation [13].

As for the complexity of the models under study, we can say that the main focus is on systems that can be described by a mathematical model with two degrees of freedom.

In the case of grinding, the non-linear factor is, for example, the cutting forces and friction forces in the contact zone between the tool and the surface to be machined (Fig. 1).

To study the properties of self-excited oscillations of the mechanical system of the MTFTP during finishing operations with one or two degrees of freedom associated with the influence of the heterogeneity of the processed material, external disturbing forces in the ranges of fundamental and subharmonic resonances containing, in addition to the self-exciting element, also the zone of dry friction of the tool with the processed surface [14].

The main types of the studied systems are shown in Fig. 1 and differ, on the one hand, in the number of degrees of freedom and, on the other hand, in the location of the self-exciting element.

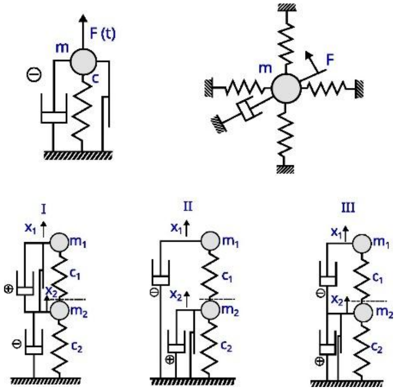



Fig. 1 Types of studied systems of MTFTP finishing operations

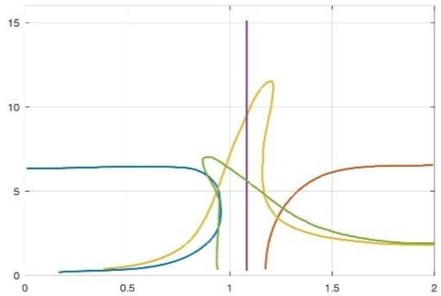



Fig. 2. An example of the behavior of a MTFTP system with one degree of freedom

In the analytical calculation of steady-state oscillations, the harmonic balance method was used, assuming a solution in the form of two independent harmonic components. The stroboscopic method based on the direct solution of the differential equations of motion was used to determine the areas of attraction.

### 3. Research objectives

- Develop mathematical models of the MTFTP system with one and two degrees of freedom, which take into account key nonlinearities - material heterogeneity, contact gaps, nonlinear damping and friction in finishing operations;
- Implement and simulate these models in the Simulink (MATLAB) environment to capture self-excited and forced vibrations under the conditions of fundamental and subharmonic resonance;
- Analyze the influence of nonlinear vibrations on surface quality indicators (roughness, burn-in, crack initiation) by extracting amplitude-frequency and phase-frequency characteristics from the simulated resonance curves;
- Apply harmonic balance and stroboscopy methods to isolate stability regions, forced vibrations, self-excited vibrations, and mixed modes, as well as to construct maps of their attraction;

- Quantify the impact of critical parameters - damping coefficients, frictional properties, excitation amplitude/frequency, gap size - on the amplitudes of oscillations and the resulting formation of surface defects;
- Propose and evaluate optimization strategies, such as the use of non-linear vibration dampers or adjusted friction coefficients, aimed at suppressing unwanted oscillations and improving the quality of the machined surface;
- Validate the models by comparing theoretical predictions with numerical simulation results (and experimental data, if available) to confirm the adequacy of the models and their applicability to real finishing processes.

## 4. Research methods

Almost all of the cases studied were verified using computational methods. Thus, the behavior of the MTFTP system in finishing operations with 1 degree of freedom is described by a differential equation with initial conditions [15], [16]:

$$\begin{gathered} \ddot{y} - (\beta - \delta y^2)\dot{y} + y + \mu y^3 = \eta^2 \cos(\eta t + \varphi) \\ y(t_0) = y_0, \qquad \ddot{y}\,(t_0) = y_{01} \end{gathered} \tag{1}$$

where $y(t)$, $\dot{y}(t)$, $\ddot{y}(t)$ are the vertical component, the rate of change, and the acceleration of nonlinear oscillations; β, δ, μ are the coefficients of inhomogeneity of the material of the treated surface characterizing the disturbing forces; η, φ are the spectral characteristics of nonlinear oscillations, and its approximate solution $y = A\cos\eta\, t + R\cos(\omega t + \varphi)$ is shown in Fig. 2.

The amplitude of self-excited oscillations within the fundamental resonances decreases to zero. In this region, the system oscillates only by forced oscillations with amplitude $A$ and frequencies η, Ω. Outside the resonance region, vibrations are created consisting of harmonic components with frequency Ω, which is confirmed by the numerical solution.

It is sometimes convenient to study the behavior of the MTFTP system in the stroboscopic phase plane. For a frequency $\eta = 0.8$, there is only a limiting cycle $L_s$ corresponding to self-excited oscillations.

At an increased frequency $\eta = 1.35$, the limiting cycle $L_s$ and the unstable center $F_n$ reappear, but a stable special point $F_s$ has appeared, which corresponds to stable forced oscillations. The separatrix $S$ limits the regions of initial conditions leading to purely forced oscillations or their combination with self-excited oscillations.

Consider solving a non-homogeneous problem with a right-hand side and a constant term $Q$:

$$\ddot{y} - (\beta - \delta y^2)\dot{y} - a y + \mu y^3 = \mathrm{Q} + \cos(\eta \mathrm{t} + \varphi) \tag{2}$$

This term is manifested in a much more complex form of resonance curves, as can be seen in Fig. 3. The solution showed more significant deviations in the subharmonic resonance region. Therefore, attention was also drawn to the mutual influence of self-excited and forced oscillations in the subharmonic resonance region of order $1/2$. These studies have brought about new information on the shapes of the resonance curves and the shape of the attraction region.

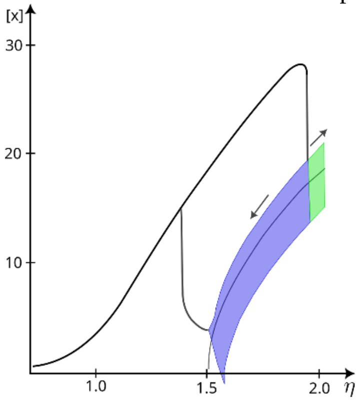



Fig. 3 Shapes of the resonance curve with a region of forced oscillations, a region of ambiguity, and a region of mixed two-frequency motion

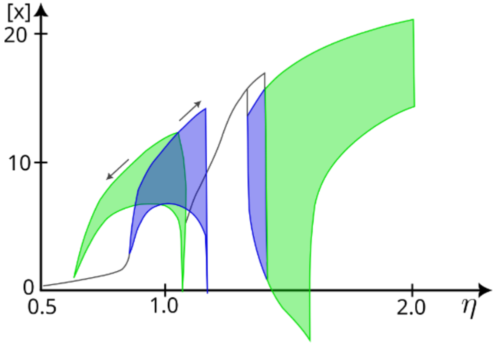



Fig. 4 Results of measurements of the change in the shape of the resonance curves at a slight change in the parameters $g$, $\varkappa$

Two-degree-of-freedom systems have been studied for two main types: on the one hand, for two-mass systems and, on the other hand, for systems with two degrees of freedom, but containing only 1 mass, the grinding wheel. The latter can, in the case of equality of linear stiffnesses in the $x$ and $y$ directions, be considered as the basic grinding model, which can include external damping, nonlinearity of restoring forces in bearings, and imbalance.

The equations of motion of a single mass case are determined by the following formulas [16]:

$$\ddot{x} + g\eta\dot{y} + \varkappa\dot{x} + \beta\eta y + [1 + \mu(x^2 + y^2)]x = \eta^2 \cos \eta t \tag{3}$$

$$\ddot{y} - g\eta\dot{x} + \varkappa\dot{y} - \beta\eta x + [1 + \mu(x^2 + y^2)]y = \eta^2 \sin \eta t \tag{4}$$

$$y(t_0) = y_0, \qquad \dot{y}(t_0) = y_{01}, \qquad x(t_0) = x_0, \qquad \dot{x}(t_0) = x_{01}$$

where $\ddot{x}$, $\dot{x}$, x, $\ddot{y}$, $\dot{y}$, $y$ are horizontal and vertical subharmonic vibrations and their dynamic components; $y_0$, $y_{01}$, $x_0$, $x_{01}$ initial conditions $g$, χ, β, μ are nonlinear characteristics of the elements of the dynamic system of the MTFTP; μ is the proportionality coefficient of the movement of the system links.

Equations (3)-(4) were solved analytically and numerically. An example of the shape of the resonance curve is shown in Fig. 3, where the regions of purely forced oscillations, the region of ambiguity, and the region of mixed two-frequency motion are visible.

The measurement results of the slightly modified case are shown in Fig. 4. It can be seen that even a slight change in the grinding wheel speed parameter causes a significant change in the shape of the resonance curves. These studies made it possible to formulate the basic laws that govern the behavior of such systems.

In this paper a similar one-mass system was considered, but with respect to grinding machines [17], [18]. As a source of self-excited oscillations, in addition to the terms $(1-x^2)\dot{x}$, $(1-y^2)\dot{y}$, we considered the assumption of a coordinate relationship, which gives rise to the equation:

$$\ddot{x} + B_1 y + n_1^2 x - \varepsilon(1-x^2)\dot{x} = 0 \quad (5)$$

$$\ddot{y} + B_2 x + n_2^2 y - \varepsilon(1-y^2)\dot{y} = 0 \quad (6)$$

and, depending on the value of the constants $B_1$, $B_2$, $n_1$, $n_2$, the equilibrium state can also be either stable or unstable at $\varepsilon = 0$. The behavior of the system (Fig. 5) is depicted in the phase plane, in which the bifurcation properties of the system can be studied, i.e. the conditions under which single-frequency or two-frequency solutions exist. In this case, it was determined how to use a dry friction damper to stabilize the equilibrium state of oscillation of a self-excited system with two degrees of freedom and thereby achieve improvements in the operational properties of grinding machines. These research results have been confirmed experimentally. Thus, works [9], [19] show the occurrence of self-excited oscillations during grinding, taking into account the influence of coordinate coupling. First, the influence of the connection between the machining parameters and the roughness of the machined surface of the workpiece and the oscillating MTFTP system was taken into account.

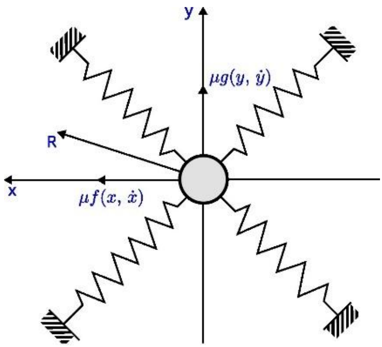



Fig. 5 Single mass system for calculating self-excited vibrations during grinding

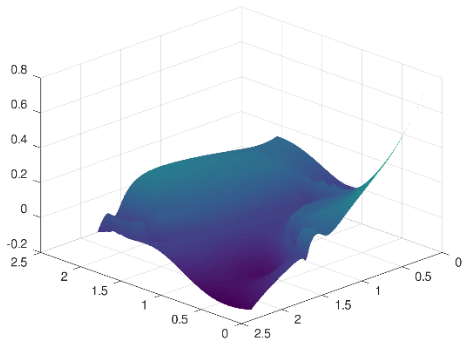



Fig. 6 An example of reduction of self-excited oscillations by changing the parameters $B_1$, $B_2$

The main means of reducing self-excited vibrations is the method of fixing the tool to an elastic base or attaching it to damping elements. This means can be applied to other elements of the MTFTP. It turns out that self-excited vibrations can be suppressed quite successfully in a wide range of system parameters. An example of reducing oscillations by changing the image parameters is shown in Fig. 6.

These studies not only demonstrate the behavior of mechanical systems capable of self-excited oscillations, but also allow their results to be successfully applied to optimize the quality characteristics of machined surfaces in finishing operations. Self-excited oscillation can be caused in mechanical MTFTP systems for a number of reasons. One of these reasons is, for example, the connection between the processes occurring in the oscillating system and the external environment.

The instability of motion in a mechanical system can also be caused by periodic changes in the internal parameters of the system.

The amplitudes of the oscillations of the studied system, in which instabilities or significantly increased amplitudes of oscillations are formed, are determined. It is shown that the dependence of the amplitude $A$ in the main resonance as a function of the frequency variation α and the cubic damping β in the region of relative motion oscillation is self-excited, and the influence of the frequency variation α on the multivalued resonant amplitude $A$ is also observed. Let us consider the case of nonlinear oscillations of the mechanical system of the MTFTP described by the system of differential equations:

$$\begin{aligned} y' + \lambda y = \varepsilon \mathrm{E}y'' + \mathrm{F}y_1'' + y_2'' + \mathrm{B}y'' + \mathrm{D}y_3' + y(\mathrm{Y}\cos \mathrm{k\,ar} + \mathrm{U}\sin \mathrm{k\,ar}) \\ + y_2 + \mathrm{I}\sin \mathrm{k\,ar} + \Phi(\mathrm{r}) \end{aligned} \qquad (7)$$

where $y$ is the vector of relative motion and motion of the supports, λ is the diagonal matrix of the system eigenvalues, ε is a small parameter, $B$, $D$, $E$, $F$, $Y$, $U$, $I$ are the matrix coefficients, and $f(t)$ is the vector of excitatory forces.

The torsional vibrations of a MTFTP mechanical system with $n$ degrees of freedom and Hooke joints were studied. The motion is described by a linearized system of equations with periodic coefficients of the following type [16]:

$$M\ddot{q} + \varepsilon B\dot{q} + Cq + \lambda(t)q = f(t) \qquad (8)$$

here ε is a small parameter; $M$, $B$, $C$ are symmetric matrices of order $n$, $\lambda(t)$ is a diagonal matrix; $q(t)$ are generalized coordinates.

From the theoretical analysis based on orthonormalized transformations, the boundary curves of the instability region and the damping value conditions that ensure stability in a given speed range were obtained.

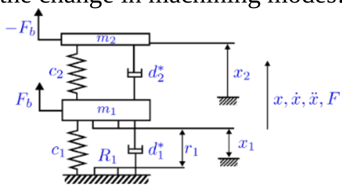
The areas of the main subharmonic and subultraharmonic resonances have been experimentally studied. The oscillation of a mechanical system is significantly complicated when shocks appear in the MTFTP system due to inhomogeneities in the surface layer of the parts to be ground or internal shocks at limited gaps in the connection of individual MTFTP elements.

These systems were analyzed using mathematical models that can be schematically represented by systems with one or two degrees of freedom, in which the limiters are arranged differently and in which the oscillations are excited either by external harmonic forces or contain self-excited terms.

A system with two degrees of freedom and one pair of restraints is shown in Fig. 7. While the impacts occur in the lower part of the system, a device is introduced into the upper part, exerting a force $F_b - F_b$ on both masses $m_1$ and $m_2$. The form of these forces is shown in Fig. 7. The flat hysteresis curve of the excitatory effects depends on the parameters of the relative motion between the masses. Depending on the initial conditions, there are different types of motion. These types of motion also depend on the magnitude of the self-exciting force $F$, on the attenuation, and on the size of the gaps $r_1$ between the elements of the MTFTP.

An example of the regions of existence and stability of different types of motion depending on the parameters $\bar{h}$ and $\bar{r} = r_1 r_2 / F_0$ is shown in Fig. 8. At the boundaries $A_1$, $A$, there is a transition from motion without shocks to motion with shocks. The inclined shaded areas correspond to the main periodic single-impact motions with the first or second form of oscillations; the horizontally shaded areas correspond to the beating type motions. It can be seen that for some combinations of $\bar{h}$, $\bar{r}$ parameters, only one type of motion can occur, and for other combinations there are several possible types of motion.

The system with two pairs of impactors is shown in Fig. 9. The static gaps between the restraints are labeled $r_1$ and $r_2$. Since the system contains two strong non-linearities, its motion is very complex. Separate types of motion differ in the number of strokes per period of the excitation force and can exist only under certain conditions. Fig. 10 shows the regions of existence for different gap sizes: $\overline{r_1} = r_1 c / F_0$ and $\overline{r_2} = r_2 c / F_0$. The symbols $z_1$ and $z_2$ represent the number of strokes in the lower and upper regions of the change in machining modes.



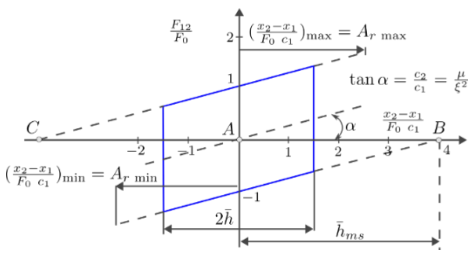



Fig. 7. The MTFTP system with two degrees of freedom and one pair of limiters

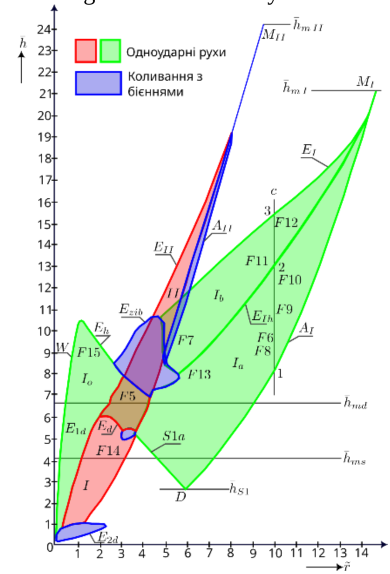



Fig. 8 An example of the region of existence and stability of different types of motion depending on the parameters $\bar{h}$ and $\bar{r} = \frac{r_1 r_2}{F_0}$

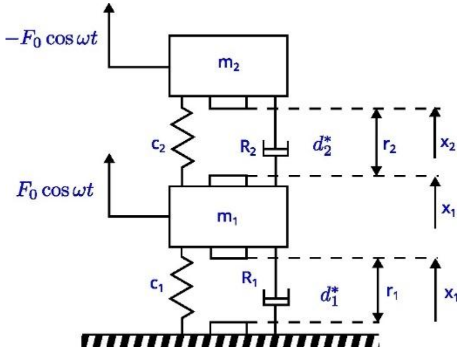



Fig. 9 A two-mass MTFTP system with static gaps $r_1$ and $r_2$ and two strong nonlinearities

These studies lead to larger-scale investigations of the effect of viscous damping and dry friction type dampers on the vibrations of systems with impacts, studies of the properties of systems containing plastic impacts, and solutions to optimal problems that allow finding the parameters of a technological system with maximum microimpact energy (for example, when the treated surface has inhomogeneities) or with minimum amplitudes in resonant regions.

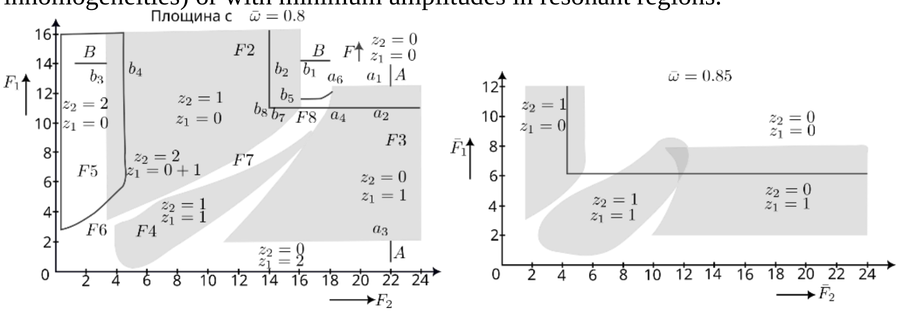



Fig. 10 Image of the region of existence at different gap sizes $\bar{r}_1 = \frac{r_1}{F_0} c$ and $\bar{r}_2 = \frac{r_2}{F_0} c$

Research on impact systems is mainly aimed at finding out the basic laws of behavior and optimization of the impact-impacted MTFTP accompanying finishing operations in terms of forming a machined surface from a heterogeneous material and its quality characteristics [14], [20], [21].

In addition to impacts, there is another type of highly nonlinear, non-analytical dependence in MTFTP mechanical systems; this is the friction zone between the tool and the machined surface. The friction forces generated in the machining zone, although much smaller than the cutting forces, have a decisive influence on oscillations in the resonant regions and on the occurrence of self-excited oscillations.

The oscillations of the MTFTP mechanical system that occur during the grinding of materials of heterogeneous structure are excited by the unevenness of the cutting forces and the friction forces. A diagram of such a studied system with two degrees of freedom is shown in Fig. 11.

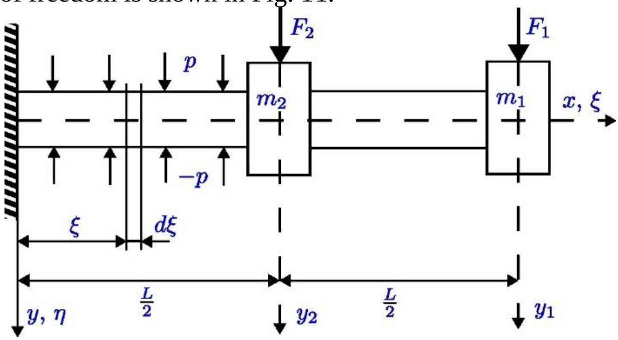



Fig. 11. Scheme of studying the MTFTP system with two degrees of freedom

When masses $m_1$ and $m_2$ move, elastic slippage occurs between two or more masses located on a beam connecting mass $m_2$ to the base. The analytical and numerical solution method based on the Krylov-Bogolyubov-Mitropolsky method allowed us to calculate the amplitude-frequency and phase-frequency characteristics of the steady-state vibrations. The equivalent attenuation and stiffness coefficients of this connection were determined for different elements of the MTFTP, the resonance characteristics under harmonic excitation were calculated, and the limiting values of forces for which it is impossible to constructively dampen vibrations at resonance. In this case, the amplitudes of oscillations grow unlimitedly, leading to the so-called resonant instability.

The example shown in Fig. 12 demonstrates the resonance amplitude curves of a system with hysteresis damping. Fig. 13 shows an example of optimizing the damping properties of a MTFTP system. With the correct choice of the friction coefficient, expressed by the parameter $\delta_0$, which is achieved by using certain lubricating cooling process media, the minimum height of the resonant peak can be achieved.

### 5. Research results

Theoretical results were verified by modeling in the Simulink system of the MATLAB package. The coincidence of theory and experiments confirmed the validity of the applied method.

The analytical solution using the statistical linearization method is supplemented and verified by calculations in MATLAB. The system of equations under study has the following form [11], [22]:

$$\ddot{y}_2 + \delta_\gamma(\dot{y}_2 - \dot{y}_1) + \gamma^2[(y_2 - y_1) + sy_0^2(y_2 - y_1)^3] = 0 \quad (9)$$

$$\ddot{y}_1 + y_1 + \nu\ddot{y}_2 = x(t) \quad (10)$$

where $x(t)$ is a random process with the character of frequency-bounded white noise.

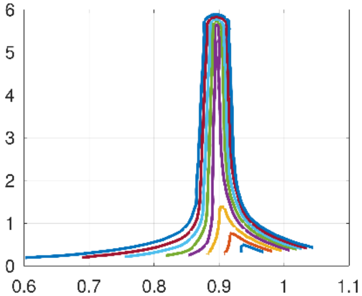



Fig. 12 Image of the amplitude resonance curves of the MTFTP system with hysteresis damping

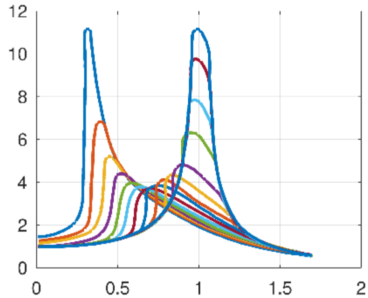



Fig. 13 Optimization of the attenuation properties of the MTFTP system due to the coefficient of friction between the tool and the machined surface

The non-linear oscillations of the systems were studied by the statistical identification method, which allowed us to determine the main characteristics of stochastically excitable systems.

When studying the properties of nonlinear systems with two degrees of freedom, attention was also drawn to the possibility of using a nonlinear vibration damper to reduce the amplitudes of a stochastically excited technological system.

It turns out that the correct choice of the nonlinearity coefficients $sy_0^2$, related to the technological conditions of finishing operations, can significantly reduce the oscillations of the MTFTP system. The dependence of the variance of the input signal $D[\dot{X}]$ and the output signal $D[\dot{Y}]$ at different nonlinearities $sy_0^2$ is shown in

Fig. 14, which can also serve as the basis for optimizing the characteristics of the nonlinear damper of stochastic oscillations of the mechanical system of the MTFTP.

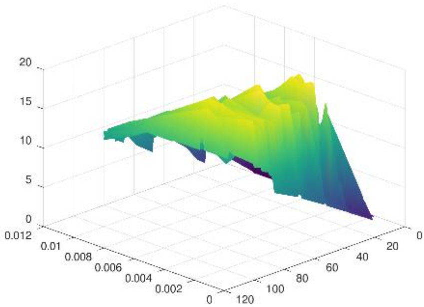


Fig. 14 Dependence of the variance of the input signal $D[\dot{X}]$ and the output signal $D[\dot{Y}]$ for different nonlinearity of $sy_0^2$ caused by technological parameters

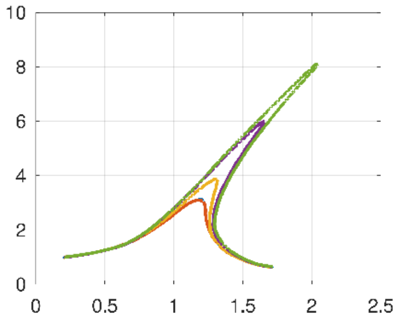


Fig. 15 Shapes of the amplitude and phase resonance curve for a system with one degree of freedom

It should be noted that the field of synthesis and identification of nonlinear systems, i.e., the compilation of the structure of a computational model, the determination of its parameters, or the characteristics of nonlinear terms based on the analysis of the motion of the mechanical system of the MTFTP, is a complex problem.

These studies have shown that it is not enough to calculate only the amplitude spectrum, but it is also necessary to determine the phase shift between the individual components of the oscillation.

Mathematical modeling of nonlinear oscillatory processes has shown that the use of identification methods for nonlinear systems is of a qualitative nature, which uses knowledge of the shape of some responses of nonlinear systems. For this reason, spectral analysis of the shape of the machined surface at the finishing stage is quite effective.

In [23], the main properties of the resonance curves expressed by the dependence of the real ($\mathrm{u} = \mathrm{a}\cos\psi$) and imaginary ($v = a\sin\psi$) parts of the response of a nonlinear dynamic system are shown. For a system with one degree of freedom, in which the shapes of the amplitude and phase resonance curves are well known (Fig. 15), the same resonance curves in the variables $u$ and $v$ have a special character (Fig. 16).

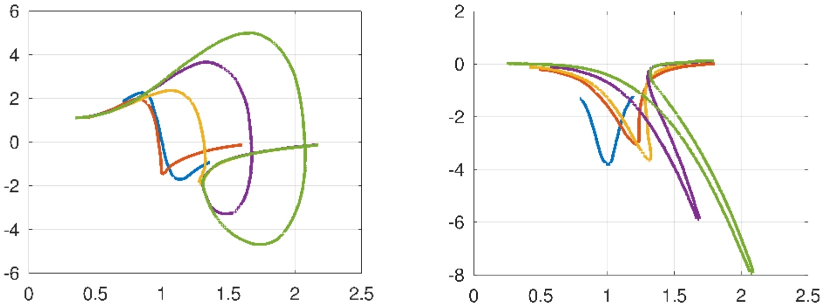



Fig. 16 Resonance curves in the variables $u$ and $v$

As the attenuation decreases, the resonant peak of the real part increases but gradually tilts. The resonance curve for two degrees of freedom has a similar shape (Fig. 17 – real component). Unstable branches are indicated by dashed lines. The results of the measurements in the Simulink system are shown in Fig. 18.

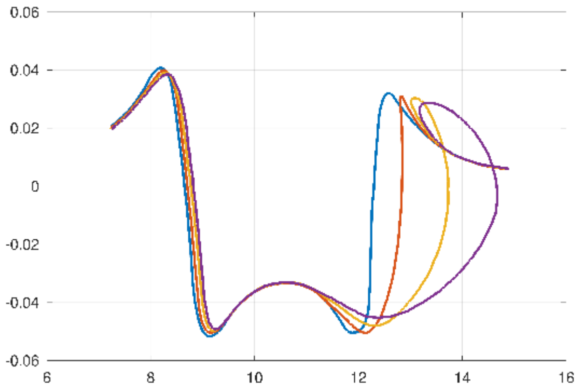



Fig. 17 Resonance curve for two degrees of freedom of the MTFTP system

The full and dashed lines indicate the resonance curves of the first and second masses in a nonlinear system with 2 degrees of freedom; the arrows indicate jumps in the resonance.

It has been shown that resonant curves during nonlinear vibrations of mechanical systems of finishing operations affect not only the formation of the roughness of the machined surface, but also the appearance of burn marks on them and the formation of defects such as cracks [11], [14], [24].

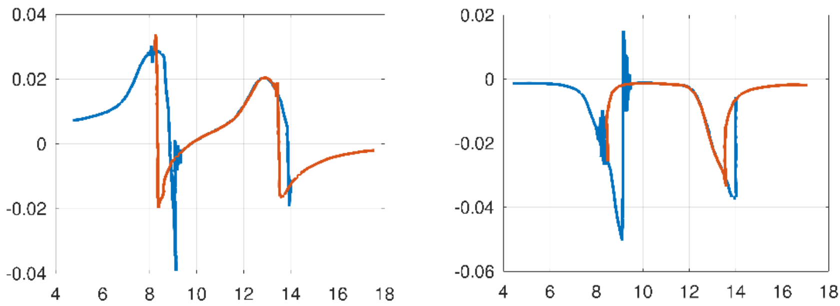


Fig. 18 Results of measuring the resonance curves in the SIMULINK system

These resonance curves are used to calculate the maximum amplitudes of forced oscillations in a given frequency domain, to determine the natural frequencies, to estimate the attenuation, and to calculate the characteristics of the nonlinear term (element) (Fig. 19).

All the conclusions that follow from the experimental study of nonlinear systems are more reliable the more accurately the oscillation parameters are measured, especially the amplitudes and phases of the fundamental or higher harmonic components.

In this case, we study, first of all, the possibilities of registering those quantities and phenomena that are typical for nonlinear oscillations.

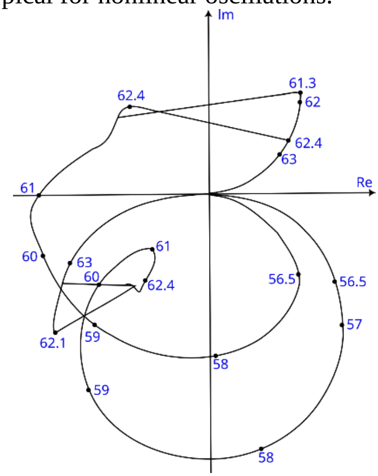


Fig. 19 Image of the resonance curves of a nonlinear system in the Kennedy-Pank diagram

Measuring instruments for oscillatory processes containing non-linear elements are practically identical to those used to measure linear systems. It is necessary, however, to pay special attention to the accuracy of measuring the amplitude and phase resonance curves, and to the possibilities of a comprehensive analysis of the response of a nonlinear system.

An example of the measured amplitude and phase resonance curves is shown in Fig. 20. For comparison, the full results of the theoretical solution are also included. A relatively good agreement of both approaches is visible. To identify nonlinear oscillations, the method of measuring oscillations of mechanical systems and their processing using Simulink was improved [25], [26].

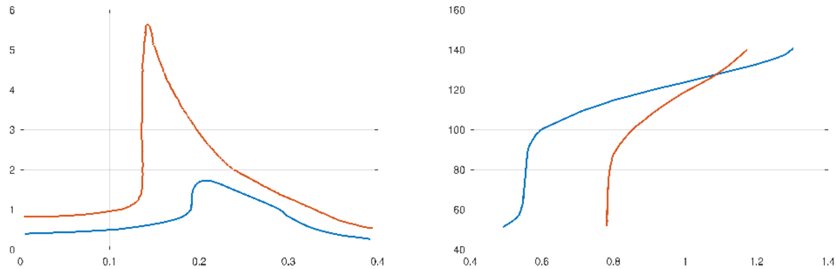



Fig. 20 (a) Theoretical and (b) experimental values of the amplitude and phase resonance curves

Since the numerical recording of measured quantities is more accurate and the numerical processing of measured indicators, for example, using Fourier analysis, is much more accurate, it can be expected that the use of the Simulink system allows for a more detailed and accurate study of the nonlinear properties of the MTFTP in finishing operations and a more accurate determination of the nonlinear characteristics of their elements.

## 6. Conclusions

The presented calculations and experimental studies of nonlinear oscillations in the mechanical system of finishing operations have shown their significant influence on the formation of qualitative characteristics when processing the working surfaces of parts made of materials containing inhomogeneities. This especially applies to resonance curves, which are used to calculate the maximum amplitudes of forced oscillations in a given frequency range, to establish natural frequencies, to estimate damping, and to calculate the characteristics of non-linear members of the mechanical system of the MTFTP.

**References:** **1.** *W. Grzesik*, ‘Influence of surface textures produced by finishing operations on their functional properties’, Journal of Machine Engineering, vol. 16, no. 1, pp. 15–23, 2016. **2.** *W. Grzesik*, ‘Influence of surface roughness on fatigue life of machine elements – the development in experimental investigations and simulations', Mechanik, vol. 5–6, pp. 307–313, May 2019, doi: 10.17814/mechanik.2019.5-6.39. **3.** *V. Lavrinenko and V. Solod*, 'The ralationship betwheen the parameters of roughness and features of surface formation with a special microprofile', Cutting & Tools in technol. syst., vol. 96, pp. 99–109, Mar. 2022, doi: 10.20998/2078-7405.2022.96.11. **4.** *I. I. Khafizov and I. G. Nurullin*, 'Influence of roughness on operational properties of details', IOP Conf. Ser.: Mater. Sci. Eng., vol. 570, no. 1, p. 012048, Jul. 2019, doi: 10.1088/1757-899X/570/1/012048. **5.** *N. Kheir*, Systems modeling and computer simulation. Routledge, 2018. **6.** *T. N. Ivanova*, 'Design and technological methods of reducing of thermal stress in grinding process', MATEC Web Conf., vol. 224, p. 01006, 2018, doi: 10.1051/matecconf/201822401006. **7.** *A. Bezpalova, V. Lebedev, N. Klimenko, T. Chumachenko, and I. Uryadnikova*, 'Investigation and analysis of the possibility of diffusionless phase transformations in the surface layer of a part under the action of grinding temperatures', EEJET, vol. 4, no. 12 (94), pp. 36–42, Aug. 2018, doi: 10.15587/1729-4061.2018.140982. **8.** *T. N. Ivanova*, 'Research of working capacity of grinding wheels', MATEC Web Conf., vol. 224, p. 01005, 2018, doi: 10.1051/matecconf/201822401005. **9.** *O. Yakimov, L. Bovnegra, S. Uminsky, V. Tonkonogyi, and Y. Shichireva*, 'Wear grinding wheels with precise working surface', Cutting & Tools in technol. syst., vol. 92, pp. 197–207, Jul. 2020, doi: 10.20998/2078-7405.2020.92.21. **10.** *V. Gusev*, 'Surface microgeometry treated with a precast textured grinding wheel', MATEC Web Conf., vol. 329, p. 03019, 2020, doi: 10.1051/matecconf/202032903019. **11.** *E. B. Belov, S. L. Leonov, and A. A. Sitnikov*, 'Forecasting of Machined Surface Waviness on the Basis of Self-Oscillations Analysis', AMM, vol. 788, pp. 35–40, Aug. 2015, doi: 10.4028/www.scientific.net/AMM.788.35. **12.** *R. Strelchuk*, 'Surface roughness modeling during electric discharge grinding with variable polarity of electrodes', Cutting & Tools in technol. syst., vol. 94, pp. 77–84, Jun. 2021, doi: 10.20998/2078-7405.2021.94.09. **13.** *Y. Menshikov*, 'Criteria for Adequacy Estimation of Mathematical Descriptions of Physical Processes', in Computational Models in Engineering, K. Volkov, Ed., IntechOpen, 2020. doi: 10.5772/intechopen.82725. **14.** *V. I. Gurei*, 'Modeling the amplitude-frequency characteristic of the friction hardening process of flat surfaces of machine parts', Bulletin of the National Technical University "KhPI". Series: Dynamics and Strength of Machines, vol. 2, pp. 30–35, Apr. 2020, doi: 10.20998/2078-9130.2020.1.219636 (in Ukrainian). **15.** *G. A. Oborsky, A. F. Dashchenko, A. V. Usov, and D. V. Dmytryshyn*, Modeling of systems: monograph. Odesa: Astroprint, 2013 (in Russian). **16.** *A. V. Usov, O. M. Dubrov, and D. V. Dmytryshyn*, Modeling of Systems with Distributed Parameters: Monograph. Odesa: Astroprint, 2002 (in Ukrainian). **17.** *T. Liu, Z. Deng, L. Lv, S. She, W. Liu, and C. Luo*, 'Experimental Analysis of Process Parameter Effects on Vibrations in the High-Speed Grinding of a Camshaft', SV-JME, vol. 66, no. 3, pp. 175–183, Mar. 2020, doi: 10.5545/sv-jme.2019.6294. **18.** *C.-C. Wang, X.-X. Zhuo, and Y.-Q. Zhu*, 'Optimization Analysis of Vibration for Grinder Spindle', Sensors and Materials, vol. 32, no. 1, p. 407, Jan. 2020, doi: 10.18494/SAM.2020.2603. **19.** *S. Dyadya, O. Kozlova, E. Kushnir, and D. Karamushka*, 'Research of oscillations during end-milling and their influence on the formation of the machined surface', HMT, vol. 2, pp. 71–77, Jan. 2023, doi: 10.15588/1607-6885-2022-2-12. **20.** *T. I. Zohdi*, 'Modeling and simulation of the post-impact trajectories of particles in oblique precision shot-peening', Comp. Part. Mech., vol. 3, no. 4, pp. 533–540, Nov. 2016, doi: 10.1007/s40571-015-0048-5. **21.** *M. Rifat, E. C. DeMeter, and S. Basu*, 'Microstructure evolution during indentation of Inconel-718 created by additive manufacturing', Materials Science and Engineering: A, vol. 781, p. 139204, Apr. 2020, doi: 10.1016/j.msea.2020.139204. **22.** *V. Fedorovich, I. Pyzhov, and I. Voloshkina*, 'Modeling of the process of vibratory grinding by finite element method', Cutting & Tools in technol. syst., vol. 90, pp. 136–150, May 2019, doi: 10.20998/2078-7405.2019.90.14. **23.** *B. Ma and Y. Ren*, 'Nonlinear Dynamic Analysis of the Cutting Process of a Nonextensible Composite Boring Bar', Shock and Vibration, vol. 2020, pp. 1–13, Oct. 2020, doi: 10.1155/2020/5971540. **24.** *F. Novikov*, 'Optimisation of interrupted grinding parameters according to the temperature criterion', Cutting & Tools in technol. syst., vol. 98, pp. 59–72, May 2023, doi: 10.20998/2078-7405.2023.98.06. **25.** *P. Sivák, I. Delyová, J. Bocko, and J. Šarloši*, 'Some methods of analysis of chaos in mechanical systems', AJME, vol. 2, no. 7, pp. 199–203, Nov. 2014, doi: 10.12691/ajme-2-7-6. **26.** *V. Nekhaev, V. Nikolaev, and M. Safronova*, 'Numerical simulation of oscillations of a nonlinear mechanical system with a spring-loaded viscous friction damper', MATEC Web Conf., vol. 265, p. 05030, 2019, doi: 10.1051/matecconf/201926505030.

Анатолій Усов, Максим Куніцин, Одеса, Україна

# МОДЕЛЮВАННЯ ВПЛИВУ НЕЛІНІЙНИХ КОЛИВАНЬ НА ЯКІСТЬ РОБОЧОЇ ПОВЕРХНІ ДЕТАЛЕЙ НА ФІНІШНИХ ОПЕРАЦІЯХ

***Анотація.*** *Досліджується вплив фінішних операцій на шорсткість оброблюваних поверхонь у випадку, коли система «Верстат-Пристосування-Інструмент-Деталь» (ВПІД) знаходиться в зоні нелінійних коливань. Для досягнення поставленої мети будуються математичні моделі динамічних коливальних процесів, що супроводжують обробку робочих поверхонь деталей, із використанням середовища Simulink пакету MATLAB. Проведене моделювання дозволяє визначити умови формування самозбуджених коливань механічної системи ВПІД під час фінішних операцій, як у випадку однієї, так і двох ступенів свободи, що виникають внаслідок неоднорідності оброблюваного матеріалу, зовнішніх збурюючих сил та їхнього впливу в діапазонах основних і субгармонічних резонансів. Враховано також наявність зон сухого тертя між інструментом і оброблюваною поверхнею, що істотно впливає на характер коливань. Наведені в роботі дослідження демонструють не тільки поведінку механічних систем, здатних до виникнення автоколивань, але й дають змогу успішно застосовувати результати моделювання для оптимізації якісних характеристик оброблюваних поверхонь на етапі фінішних операцій. Особлива увага приділена аналізу резонансних кривих, які при наявності нелінійних коливань у механічних системах фінішної обробки впливають не лише на формування шорсткості обробленої поверхні, але й сприяють виникненню дефектів, таких як припіки, мікротріщини та інші пошкодження поверхневого шару матеріалу. Результати моделювання показали, що характер автоколивань і резонансних явищ істотно залежить від параметрів з'єднань, властивостей оброблюваного матеріалу, а також від умов зовнішнього навантаження. Це дозволяє запропонувати методи регулювання процесу обробки через зміну технологічних режимів, зокрема використання демпфувальних елементів або контрольованого сухого тертя для зменшення амплітуд небажаних коливань. Отримані дані можуть бути використані для вдосконалення конструкції елементів технологічного оснащення, підвищення точності обробки та поліпшення експлуатаційних характеристик деталей машин.*

**Ключові слова**: *фінішні операції; нелінійні коливання; моделі; система SIMULINK; амплітуди коливань; стійкість.*