

## QUALITY OF MACHINED SURFACES OF THIN-WALLED ELEMENTS OF PARTS UNDER THE ACTION OF REGENERATIVE VIBRATIONS DURING MILLING WITH END MILLS

Sergei **Dyadya** [0000-0002-7457-7772], Olena **Kozlova** [0000-0002-3478-5913], Pavlo **Tryshyn** [0000-0002-3301-5124], Eduard **Kondratiuk** [0000-0002-2672-7174], Oleksii **Kramarenko** [0009-0003-4628-853X]

National University «Zaporizhzhya Polytechnic», Zaporizhzhya, Ukraine  
[kozlova@zntu.edu.ua](mailto:kozlova@zntu.edu.ua)

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**Abstract.** *Monowheels, in which the disk and working blades constitute a single structure without detachable connections, are critically important components of new generation gas turbine engines for aviation and power engineering. The use of such structures allows to reduce the weight and increase the reliability of the units. However, errors in the shape of the functional surfaces of the blades that arise during mechanical processing lead to a decrease in the engine efficiency and an increase in fuel consumption. To ensure the accuracy of manufacturing the functional surfaces of the blades during the technological process, it is necessary to know the conditions under which cutting occurs. The work uses an approach based on the distribution of the entire range of cutting speeds into five speed oscillation zones. It was established that the third speed zone is the most unfavorable during milling of blades on multi-coordinate CNC machines. It is characterized by high-intensity vibrations, which worsens the quality of the machined surface and reduces the stability of the cutter. Their cause is regenerative oscillations, which are excited during machining by the trace left on the cutting surface by the accompanying free oscillation. The purpose of the study is to determine the influence of the axial depth of cut on the quality of the machined surface during milling in the third speed zone of vibrations. The results of the experiments showed that due to the peculiarities of up-milling with an axial depth of cut of  $a_p = 1$  mm, waviness on the machined surface is not formed, even in the presence of regenerative oscillations. The obtained data allow us to develop recommendations for the selection of cutting modes that minimize the amplitude of oscillations and ensure that the monowheel blades meet high operational requirements.*

**Keywords:** *milling; end-mill; cutting surface; waviness; regenerative oscillation; forced vibrations; accompanying free oscillation.*

### 1. Introduction

Ensuring the accuracy of high-tech products, such as gas turbine engines and power plants, and the productivity of their production are urgent tasks in the aerospace and energy industries. A characteristic feature of new generation gas turbine engines is the use of monowheels in compressors and turbines. The monowheel is a single design of the disk and blades without detachable connections. The error in the shape of the complex-profile functional surfaces of the blades

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reduces the efficiency of the engine and increases fuel consumption. Because of this, much attention is paid to the conditions in which they are processed. Thin-walled elements of the blades, which are located on a massive hub, are very sensitive to vibrations that arise during surface milling and affect the accuracy of its shape and roughness [1, 2]. Vibration-free milling conditions are achieved by selecting spindle speeds and cutting depths using stability lobe diagrams [3–5] or by suppressing vibrations with tools with special geometry [6].

Analysis of the influence of vibrations on the quality of the machined surface shows that the greatest contribution is made by regenerative vibrations [7, 8], which arise due to the phase difference between the waves on the cutting surface on the current and previous rotation of the part or tool pass. It should be noted that regenerative vibrations do not always act, but only in cases where there is waviness on the cutting surface. To determine such conditions during milling with end mills, the division of the cutting speed range into five speed vibration zones is used [9]. In the first, fourth and fifth speed vibration zones, only forced vibrations act. There is no waviness on the cutting surface, so there is no reason for the occurrence of regenerative vibrations. It is for these speed zones that stability lobe diagrams are suitable, when the assigned cutting speed is coordinated according to the ratio of the frequency of free oscillations of the part to the frequency of forced oscillations [6]. During milling in the second and third speed oscillation zones, when the cutter is cut into the part, forced oscillations are excited, which are superimposed by accompanying free oscillations (AFO) [10], which act for a short time [11]. However, during milling of thin-walled elements of the part (TWE) with end mills, which include monowheel blades, this time is enough to form waviness on the cutting surface [12]. It contributes to the excitation of regenerative oscillations. At the same time, their greatest intensity falls on the third speed zone. To ensure stable milling in the second speed oscillation zone, a concomitant feed direction is used. Milling under this condition has a damping property at the beginning of cutting due to the greatest thickness of the cut layer, which suppresses the occurrence of accompanying free vibrations and the formation of waviness on the cutting surface. Therefore, the current task remains to ensure the accuracy of manufacturing parts during milling in the third speed zone of oscillations. One of the directions of its solution is to determine the optimal cutting modes. This article shows how the axial depth of cut affects the intensity of regenerative vibrations and the formation of the machined surface during up- and down-milling of the TWE by an end mill in the third speed zone of oscillations.

## **2. Research methodology**

The research was conducted on a special stand [13], where the oscillator is an elastic plate with a processed sample, which models a thin-walled element of a part

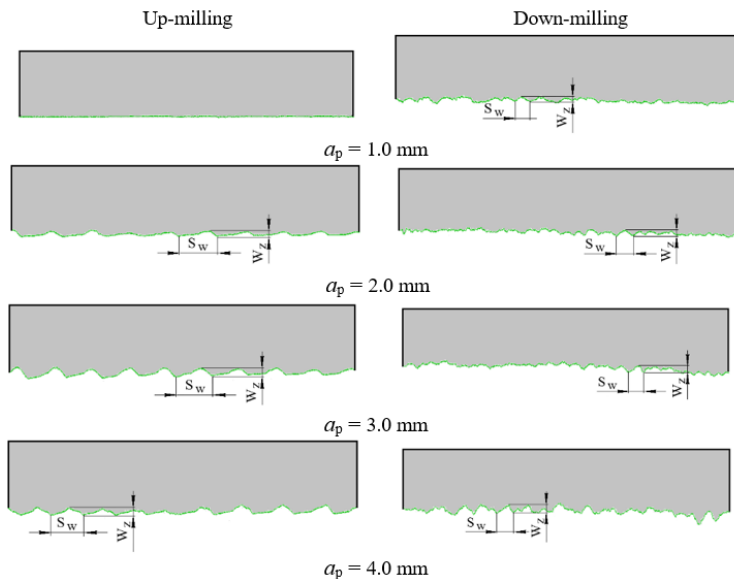
with one degree of freedom. This simplification allows us to study the most unfavorable conditions during the action of the first mode of oscillations, which are bending in shape, have the highest intensity of action and the lowest frequency of natural oscillations. To record oscillations, the sensor (proximeter XS1M18AB120) was fixed in the stand and located in the middle of the elastic plate. The signal from its oscillations during cutting was digitized using an analog-to-digital converter and recorded on a computer. The processing of samples from St3, was performed with a carbide milling cutter from VK8 according to the initial data given in Table 1. The cutting speed was chosen such that milling took place in the third speed zone of oscillations, when regenerative oscillations act. The reliability of the obtained results was checked by variance analysis with an assessment according to the Fisher criterion [14].

Table 1 - Input data for studying the influence of axial depth of cut on the laws of motion of the TWE during up- and down-milling

Frequency/period of free oscillations of the elastic element with the sample in the feed directions, $f/T$ , Hz/s	415/2.4 · 10 <sup>-3</sup> – up-milling 415/2.4 · 10 <sup>-3</sup> – down-milling
Milling cutter stiffness, $j$ , N/m	21805555
Frequency/period of free oscillations of the milling cutter, $f/T$ , Hz/s	833/1.2 · 10 <sup>-3</sup>
Milling cutter diameter, $d$ , mm	50
Number of milling cutter-teeth, $z$	1
Rake angle of the milling cutter tooth, deg	0
Flank angle of the milling cutter tooth, deg	10
Angle of inclination of the milling cutter teeth, $\omega$ , deg	0
Feed per tooth, $S_z$ , mm	0.1
Spindle speed, $n$ , rpm	280
Radial cutting depth, $a_e$ , mm	0.5
Axial cutting depth, $a_p$ , mm	1; 2; 3; 4

### 3. Experiments and discussion of results

After milling, profilograms were recorded (Fig. 1), which determined the pitch –  $S_w$  and the waviness height –  $W_z$  of the machined surfaces, the values of which are given in Table 2.



$S_w, W_z$  – pitch and height of waviness on the machined surface of the TWE

Figure 1 – Profiles of machined surfaces after up- and down-milling of the TWE with different axial cutting depths

Table 2 – Pitch and height of waviness on machined surfaces after up- and down-milling of TWE with different axial depths of cut

Axial depth of cut $a_p$ , mm	Waviness pitch, $S_w$ , mm		Waviness height, $W_z$ , mm	
	Up- milling	Down-milling	Up- milling	Down-milling
1.0	-	1.14	-	0.054
2.0	2.78	1.09	0.055	0.060
3.0	2.72	1.05	0.086	0.065
4.0	2.68	1.23	0.096	0.105

On the profilegraphs obtained after milling, waviness is observed on all machined surfaces, except for the surface after up-milling with an axial cutting depth of 1 mm. To determine the influence of the axial cutting depth on the pitch and height of waviness on the machined surfaces, an analysis of variance was performed with an assessment using the Fisher criterion. The calculation results are given in Tables 3-6. The Fisher criterion determined by the results of measuring the waviness pitch

is less than the critical value. This indicates that the axial cutting depth does not affect the waviness step during up- and down milling.

Table 3 – One-way analysis of variance of the influence of axial depth of cut on the waviness pitch of machined TWE surfaces during up-milling

Sum of squares		Number of degrees of freedom		Variance		Fisher's exact test	
$Q_A$	$Q_W$	$f_A$	$f_W$	$S_A^2$	$S_W^2$	$F$	$F_{cr}$
0.050	1.405	2	27	0.025	0.052	2.08	3.354

Table 4 – One-factor analysis of variance of the influence of axial depth of cut on the waviness height of machined TWE surfaces during up-milling

Sum of squares		Number of degrees of freedom		Variance		Fisher's exact test	
$Q_A$	$Q_W$	$f_A$	$f_W$	$S_A^2$	$S_W^2$	$F$	$F_{cr}$
0,010	0,007	2	30	0,005	0,00023	21,74	3,316

Table 5 – One-way analysis of variance of the influence of axial depth of cut on the waviness pitch of machined TWE surfaces during down-cutting

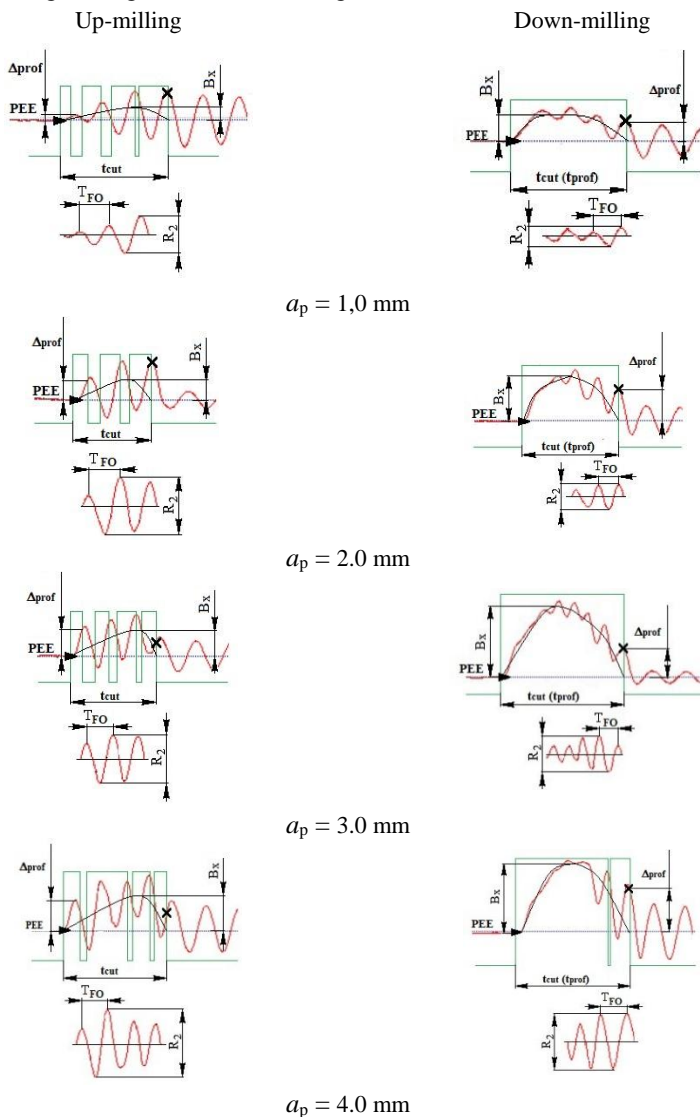
Sum of squares		Number of degrees of freedom		Variance		Fisher's exact test	
$Q_A$	$Q_W$	$f_A$	$f_W$	$S_A^2$	$S_W^2$	$F$	$F_{cr}$
0.128	0.734	3	23	0.0427	0.0319	1.338	3.028

Table 6 – One-way analysis of variance of the influence of axial depth of cut on the waviness height of machined TWE surfaces during down-cutting

Sum of squares		Number of degrees of freedom		Variance		Fisher's exact test	
$Q_A$	$Q_W$	$f_A$	$f_W$	$S_A^2$	$S_W^2$	$F$	$F_{cr}$
0.019	0.008	3	40	0.00633	0.0002	31.65	2.827

The Fisher criterion determined by the results of measuring the waviness height is greater than the critical value. This indicates that the axial depth of cut affects the waviness height during up- and down milling. The results shown in Table 2 confirm these conclusions.

To study the waviness on the cutting surfaces, which causes regenerative vibrations, fragments of oscillograms were considered, on which the vibrations of the part during cutting were recorded (Fig. 2).

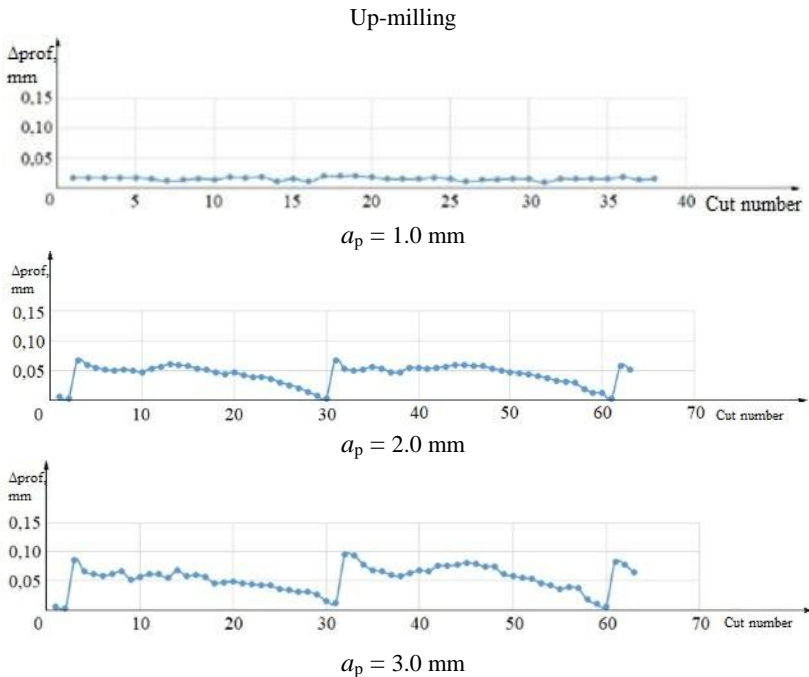


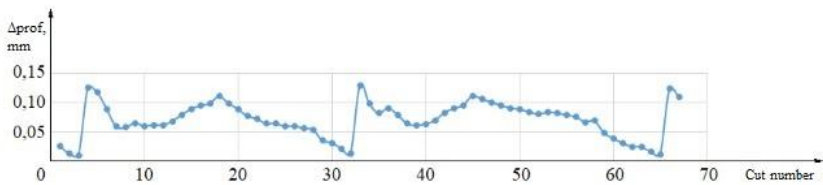
PEE – position of elastic equilibrium;  $T_{FO}$  – period of the AFO;  $R_2$  – AFO range;  $t_{cut}$  – cutting time;  $B_x$  – maximum static displacement of the TWE from PEE;  $\Delta_{prof}$  – deflection from PEE

of the first wave of the TWE during up-milling and the last wave of the TWE during down-milling

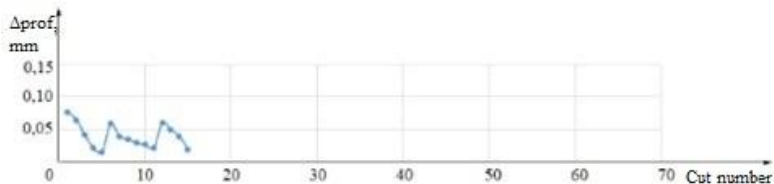
Figure 2 – Fragments of oscillograms of the oscillations of TWE obtained during up- and down-milling with different axial depths of cut

The recorded oscillation oscillograms of the thin-walled element of the part during cutting are identical in shape to the cutting surfaces [15]. Based on this, it can be said that the waviness on the cutting surfaces is present at all axial depths of cutting, which contributes to the excitation of regenerative oscillations. To determine their influence on the machined surface, graphs of changes in the deviation deflection from the initial equilibrium position (PEE)  $\Delta_{prof}$  of the first wave of the AFO during up-milling and the last wave of the AFO during down- milling were plotted (Fig. 3).

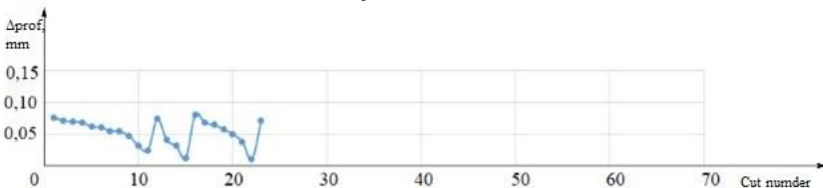




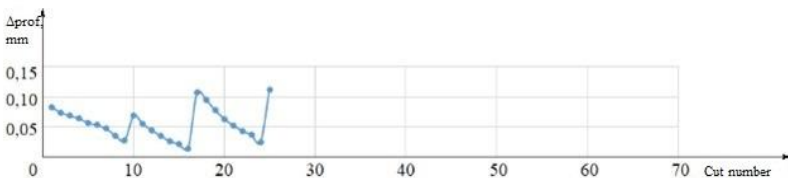
$a_p = 4.0 \text{ mm}$   
Down-milling



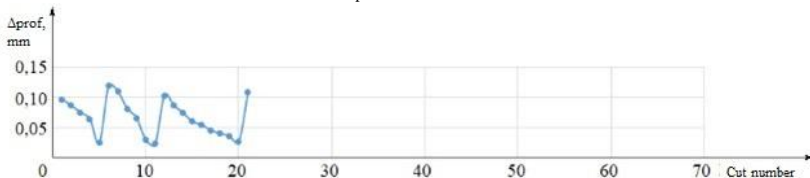
$a_p = 1.0 \text{ mm}$



$a_p = 2.0 \text{ mm}$



$a_p = 3.0 \text{ mm}$



$a_p = 4.0 \text{ mm}$

Figure 3 –Deflection from the PEE of the first wave of the AFO -  $\Delta_{prof}$  during up-milling and the last wave of the AFO -  $\Delta_{prof}$  during down-milling of the TWE with different axial cutting depths

These deflection characterize the marks on the machined surface after moving the sample by the feed rate per tooth. The graphs show that the deflection from the PEE of the first wave of the AFO during up-milling and the last wave of the AFO during down-milling have a periodicity of changes and increase with increasing axial depth of cut, except for the values obtained during up-milling with an axial depth of cut of 1 mm. This is facilitated by the peculiarities of up- and down-milling, which determine the influence of regenerative oscillations on the formation. During up-milling, it occurs at the beginning of cutting, when regenerative oscillations have not yet arisen. It is precisely because of the identical conditions for cutting the allowance during cutting with an axial depth of 1 mm that marks of the same depth remain on the machined surface and waviness is absent. With an axial cutting depth of more than  $a_p = 1$  mm during up-milling, the height of the waviness at the beginning of the cutting surface increases from the AFO, which affects the change in the depth of the marks that remain on the machined surface and form waviness on it. Its height increases with increasing axial depth. During up-milling, surface generation occurs at the end of cutting, when the regenerative vibrations are already excited during the movement of the cutter along the wave trace and the conditions change during cutting the allowance, due to which the marks that remain have different depths and form waviness on the machined surface at all axial cutting depths. Such features of the formation of the machined TWE surface must be taken into account when choosing the feed direction and assigning the axial cutting depth to ensure the accuracy of manufacturing complex-profile functional surfaces of the blades of monowheels during milling with end mills in the third high-speed oscillation zone.

#### **4. Conclusions**

Milling of thin-walled elements of parts by an end mill in the third speed zone of oscillations occurs with regenerative oscillations, which are excited when moving along a wavy trace formed on the cutting surface from accompanying free oscillations, and affect the stability of the tool and the quality of the machined surface. When assigning the axial cutting depth under such conditions, it should be taken into account that during up-milling, regenerative oscillations are excited after the tool passes through the surface generation zone, and during down-milling, their effect covers the area. Therefore, preference is given to down-milling. In this case, the axial cutting depth is assigned to the minimum so that during cutting the allowance, the conditions for surface generation the machined surface are the same, in which depressions of the same depth remain on it and there is no shape error in the form of waviness.

The results obtained can be used to substantiate rational cutting modes to ensure the accuracy of the functional surfaces of complex-profile thin-walled

monowheel blades under the action of regenerative vibrations during milling with an end mill.

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Сергій Дядя, Олена Козлова, Павло Тришин, Едуард Кондратюк,  
Олексій Крамаренко, Запоріжжя, Україна

## **ЯКІСТЬ ОБРОБЛЕНИХ ПОВЕРХОНЬ ТОНКОСТІННИХ ЕЛЕМЕНТІВ ДЕТАЛЕЙ В УМОВАХ ДІЇ РЕГЕНЕРАТИВНИХ КОЛИВАНЬ ПРИ ФРЕЗУВАННІ КІНЦЕВИМИ ФРЕЗАМИ**

**Анотація.** Моноколеса, у яких диск та робочі лопатки становлять єдину конструкцію без рознімних з'єднань, є критично важливими компонентами газотурбінних двигунів нового покоління для авіації та енергетики. Використання таких конструкцій дозволяє знизити вагу та підвищити надійність агрегатів. Однак похибки форми функціональних поверхонь лопаток, що виникають під час механічного оброблення, призводять до зниження коефіцієнта корисної дії двигуна та зростання витрат пального. Для забезпечення точності виготовлення функціональних поверхонь лопаток впродовж технологічного процесу необхідно знати умови, за яких відбувається різання. У роботі використано підхід, що базується на розподілі всього діапазону швидкостей різання на п'ять швидкісних зон коливань. Встановлено, що найбільш несприятливою під час фрезерування лопаток на багатокординатних верстатах з ЧПК є третя швидкісна зона. Вона характеризується вібраціями високої інтенсивності, що погіршує якість обробленої поверхні та знижує стійкість фрези. Їх причиною є регенеративні коливання, які збуджуються при обробленні по сліду, що залишається на поверхні різання від супроводжуючих вільних коливань. Метою дослідження є визначення впливу осьової глибини різання на якість обробленої поверхні під час фрезерування у третій швидкісній зоні коливань. Результати експериментів показали, що завдяки особливостям зустрічного фрезерування з осьовою глибиною різання  $a_p = 1$  мм хвилястість на обробленій поверхні не утворюється, навіть за наявності регенеративних коливань. Отримані дані дозволяють розробити рекомендації щодо вибору режимів різання, які мінімізують амплітуду коливань та забезпечують відповідність лопаток моноколеса високим експлуатаційним вимогам.

**Ключові слова:** фрезерування; кінцева фреза; поверхня різання; хвилястість; регенеративні коливання; вимушені коливання; супроводжуючі вільні коливання.